## This item is the archived peer-reviewed author-version of:

Simon Stevin as a central figure in the development of abstract algebra and generic programming

## Reference:

Gielis Johan.- Simon Stevin as a central figure in the development of abstract algebra and generic programming

# SIMON STEVIN AS A CENTRAL FIGURE IN THE DEVELOPMENT OF ABSTRACT ALGEBRA AND GENERIC PROGRAMMING 

Johan Gielis

Department of Bioengineering Sciences, University of Antwerp, Wilrijk-Antwerpen, Belgium
E-mail: johan.gielis@gmail.com
ORCID: 0000-0002-4536-3839


#### Abstract

Simon Stevin (1548-1620) is mainly known for the decimal system and his Clootkrans proof. His influence is also profound in infinitesimal calculus, mechanics, and even in abstract algebra and today's conception of polynomials, algorithms, and generic programming. Here we review his influence as assessed in generic programming. According to Dr. Stepanov, one of the most influential researchers in generic programming, Stevin's work on polynomials can be regarded as the essence of generic programming: an algorithm from one domain can be applied in another similar domain.


Keywords: Stevin, algebra, algorithms, generic programming

## 1. SYMMETRY AND THE LAW OF THE LEVER

One of the simplest examples of symmetry is expressed in Archimedes' Law of the Lever. For a scale or balance to be in equilibrium, the weight on both sides needs to be equal. In this case the arms of the scale are of equal length. When the arms are of unequal length, even the earth might be moved (Figure 1 left). Pappus of Alexandria (Synagoge, Book VIII) quotes Archimedes as saying: "Give me a place to stand on, and I will move the earth."


Figure 1: left: moving the earth. Right: Archimedes' Method.
The famous finding of Archimedes relating spheres, cone and cylinder can be shown in this way (Figure 1 right). This is Archimedes' Method, not his proof. Nevertheless, with this fundamental law most ideas in mathematics and science can be explained in a unified and didactical way. Equality means that a scale with arms of equal length is in equilibrium when the arms are perfectly horizontal. Inequality mean imbalance; to achieve equilibrium, it should suffice to add (or subtract) something on the other side or change the length of the the arms. For strict inequalities equilibrium cannot be achieved. For example, in the classical inequality $G M<A M$ is the best example, with the geometric mean $G M$ strictly smaller than the arithmetic mean $A M$ between two positive numbers $a$ and $b$, or $\sqrt{a b}<\frac{a+b}{2}$.

This can be used to understand the rules of arithmetic: for two integers $a$ and $b$, a square with side $a+b$ is placed on one side of the scale, and to ensure equilibrium, on the other side two squares (one of side $a$ and area $a^{2}$, and one with side $b$ and area $b^{2}$ ) and two rectangles with length $a$ and width $b$ and area $a b$ are needed. Indeed, $(a+b)^{2}=a^{2}+$ $2 a b+b^{2}$. Pythagoras Theorem $a^{2}+b^{2}=c^{2}$ provides another example. On the left side of the scale, we place two squares $a^{2}$ and $b^{2}$, and on the right side we place the square applied on the oblique side $c^{2}$. Obviously, because of the symmetry left and right are interchangeable. For three integers, $a, b, c$ this gives rise to famous Pythagorean triples, of which there are infinitely many. For two integers a and b, one can also use cubes and beams on either sides of a scale, to show that $(a+b)^{3}=a^{3}+3 a^{2} b+3 a b^{2}+b^{3}$. Squares, rectangles, cubes and beams need not have weight, if we use the scale in an abstract way.

Generalizing this beyond selected integers only, a circle with radius $R$ is described by the equation $x^{2}+y^{2}=R^{2}$. On one side of the scale with equal arms, a square with side $R$ is placed and on the other side squares with sides $x$ and $y$. There is a definite relation between $x$ and $y$, namely, that the sum of the squares is exactly equal to $R^{2}$. For this to
happen, $x$ and $y$, numbers between 0 and $R$ (their minimum and maximum values, respectively) need to be in a proportion: when the one gets closer to $R$, then the other one gets closer to zero. This provides an excellent way for pupils to learn and understand all of trigonometry and the Pythagorean relation $\cos (\theta)^{2}+\sin (\theta)^{2}=1$.

In fact, any of the conic sections can be understood in this way. Using scaling factors $a x^{2}+b y^{2}=1$ will explain ellipses, with $a$ and $b$ relating width and length of the ellipse. Using subtraction instead of addition yields $a x^{2}-b y^{2}=1$, the equation of the hyperbola. To execute this on a scale, the part with the minus sign needs to be moved to the other side, with the square of side 1 and area 1. A parabola $y=x^{2}$ can be understood as a machine that turns rectangles into squares keeping the area as an invariant $y \cdot 1=$ $x \cdot x$. Hence, on one side of the equal arm scale, we place or hang a rectangle with sides $y$ and 1 and on the other side a square with side $x$, and we are certain that the scale is in equilibrium. Whenever a certain value for $x$ is chosen, the value for $y$ has to be adapted in proportion. Hooke's Law $F=k \cdot \vec{x}$ is another simple example, whereby the extension of a spring (denoted by $\vec{x}$ ) is equal to the force $F$ exerted on the spring, and the length of the arms of the scale (or the ratio between left and right arms) is given by the spring constant $k$. Another example from physics is Kepler's Law of Periods, which states that the square of the orbital period of a planet is directly proportional to the cube of the semimajor axis of the elliptical orbit.

$$
T_{1}^{2}=\frac{4 \pi^{2}}{G\left(M_{1}+M_{2}\right)} a_{1}^{3} \quad \text { Equation } 1
$$

So the volume of a beam formed by the orbital period $T_{1}$ and the unit element is [ $T_{1} \cdot T_{1} \cdot 1$ ] (i.e. the volume of a beam with height and length $T_{1}$ and width 1), equals the volume of a cube $\left[a_{1}, a_{1}, a_{1}\right]$ (with sides equal to the semi-major axis $a_{1}$ ) up to a constant. On a weighing balance the constant can be interpreted as the length of the arms.


Figure 2: Kepler's Law of Equal Areas.

In fact, all laws of nature can be understood in this simple way. Moreover, the real power of the Law of the Lever is not so much in connecting quantities of the same kind (areas or volumes), but it allows to understand how very different quantities are connected like in the case of force and the extension of a spring, or a measurement of time it takes for a planet to complete a cycle around the sun and a characteristic of the shape of the orbit of a planet.

This is in my opinion the most fundamental legacy of Ancient Greek mathematics, namely finding common rulers to measure two incommensurable quantities in the same way. Archimedes' Law of the Lever unified all of this. All our sciences, all the laws of nature (see Pickover, 2008 for a comprehensive overview) are examples of this general Law.

## 2. THE ART OF WEIGHING

In the examples above, we have focused on combining quantities of the same "size", either as linear relationships or as quadratic ones. Only in the case of Kepler's law, the two quantities scale in a different dimension, namely a square for the semimajor axis (area) and a cube for the orbital period (volume). On one side of the scale, we place a cube with the semi-major axis as the side and on the other a beam of with width and length $T_{1}$, and height 1 . Indeed, a cube with volume $\left[a_{1} \cdot a_{1} \cdot a_{1}\right]$ and a beam with volume [ $T_{1} \cdot T_{1} .1$ ] can be balanced on the scale when the arms are adjusted with the appropriate constant in Equation 1.

The power of the neutral element is not so much making it disappear, as students learn when simplifying equations. Its real power is knowing it is there, all the time, and can be called upon when needed. Using a square with side $s$ and rectangle with lengths $l$ and width $w$, on a scale requires abstraction because square and rectangles have no weight, only area. However, using the unit or neutral element they can easily be turned into real life objects with weight, volumes $[s \cdot s \cdot 1]$ and $[l \cdot w \cdot 1]$.

An expression of the type $x^{3}+x^{2}+x$ seems to make little sense geometrically, because we add a cube, a square and a line. Using the unit element $(x \cdot x \cdot x)+(x \cdot x \cdot 1)+(x$. $1 \cdot 1)$ we add cubes and beams of the same dimension.

Can the same didactic method be used beyond lengths, areas, and volumes? For example,
in the case of relations of the type $a^{4}+b^{4}=c^{4}$ (this translates into supercircles with equations of the type $x^{n}+y^{n}=R^{n}$ or power laws of the type $y=x^{n}$ ? The latter are superparabolas (Figure 3) and are reported everywhere in science. Typical power laws in biology (describing allometry as opposed to isometry whereby two variables scale with the same power $x=y$ ) are of the type $y=x^{2 / 3}$ or $y=x^{3 / 4}$. The first case it can be rewritten as $y^{3}=x^{2}$ or as $[y \cdot y \cdot y]=[x \cdot x \cdot 1]$. In the second case however, the $x$ variable scales with the cube whereas the $y$ variable scales with a fourth power. Likewise, in the expansion $(a+b)^{4}=a^{4}+4 a^{3} b+6 a^{2} b^{2}+4 a b^{3}+b^{4}$, we have all fourdimensional cubes and beams. This can be extended for any $n$ and $m: y=x^{n / m}$ (Figure 3).


Figure 3: Superparabola's for integer values and translation into straight lines using logarithms.
This is simple: a cube with side 2 with volume $8=2^{3}$, and $2^{4}$ is simply two cubes with side 2 , namely $2 \cdot\left(2^{3}\right)$ with total volume $V=2 \cdot 2^{3}=16$. Four such cubes are $2 \cdot 2 \cdot\left(2^{3}\right)=2^{5}=32$ ). For all sides equal to 1 , the results remain the same for each power. It is the neutral element and each power of 1 always gives the same result 1 .

For the four-dimensional volume, it is not unimportant to recall how Blaise Pascal (16231662) thought about the fourth dimension: "Et l'on ne doit pas être blessé par cet quatrième dimension" (Bosmans, 1923). Which means that intelligent people should not be put off by something like the fourth dimension, because in reality it is about multiplication. And geometrically you can go in different directions. Figure 4 are geometric numbers as found in the books of Simon Stevin. His approach to geometric numbers was directly related to arithmetic, and in Definition XXXI in his book Wisconstige Ghedachten, he states that any number can be the measure of a square, cube, etc., or also that $n$-th power roots are numbers: "Que nombres quelconques peuvent estre nombres quarrez, cubiques etc. Aussi que racigne quelconque est nombre".

From this, Stevin reaches the fundamental conclusion that there are no absurd, irrational, irregular, inexplicable or mutually unmeasurable numbers, "Qu'il ny aucuns nombres absurdes, irrationels, irreguliers, inexplicables ou sourds." Nor are there any absurd, irrational, irregular, inexplicable or mutually immeasurable geometric numbers. He states that "this completes the description of the basis of the Geometric numbers, by which we hope to demonstrate their true properties, and legitimately refute some common absurdities." Simon Stevin was one of the first to state that unity was a number: "Que l'Unité est Nombre".


Figure 4: Stevin's geometric numbers.

Pascal wrote that this pulsating thinking, both geometric and algebraic, and thinking about cubes, numbers, and roots in different ways, is an art that should be practiced in our age of specialization. While the Greek thought of common rulers for incommensurable quantities, this presents a further major step.

## 3. WONDER EN IS GHEEN WONDER

For Stevin, miracles were no longer miracles if phenomena could be explained rationally, that is, geometrically. Stevin's motto (and epitaph; Feynman et al., 1963) was Magic is not magic (Devreese \& Vanden Berghe, 2008). At the interface between algebra and geometry, Stevin was first and foremost a geometer. In geometric numbers, he thought of powers in a very practical way. Simon Stevin (1548-1620), a contemporary of Galileo Galilei and Johannes Kepler, was a great admirer of Archimedes, and Stevin's vision was entirely consistent with rational mechanics, in which equilibrium and balancing are crucial (Figure 5).


Figure 5: Scales and lever in De Weeghdaet by Simon Stevin.

In 1586, the books De Beghinselen der Weeghconst (The Art of Weighing) and De Weeghdaet (The Practice of Weighing) provided the foundations of mathematical vector calculus with the rule of the parallelogram for the addition of forces as a concrete application in physics (Verstraelen, 2014).

With the famous Clootkrans proof (Figure 6), he introduced the impossibility of a perpetual motion machine as a method of proof in physics. It is discussed in his famous Feynman Lectures on Physics (Feynman, et al., 1963). In The History of Mechanics (Mach 1893) Stevin's thought experiment is considered as one of the finest in the history of science. Further, Stevin transformed the method of weighing that had inspired Archimedes into a method of proof, with the use of limit values as the culmination (Figure 6, right; Bosmans, 1926; Sarton, 1934).

Stevin's books were translated and edited by Snellius and Albert Girard and were available in Dutch, French, and Latin and were known to Gregory of Saint Vincent and Descartes, among others. Essential in Stevin's work is the relationship between Spiegeling ("theory") and Daet ("practice"). In addition to the necessary theoretical approach, there should always be an experimental approach, either concrete or through a thought experiment. In this way Stevin made valuable contributions in the fields of calculus, algebra, geometry, mechanics, hydrostatics, navigation, tidal theory, fortification, lock construction, economics, ... On the theoretical side, he also solved the hydrostatic paradox before Blaise Pascal, to whom this result is generally attributed.


Figure 6: Left: Title page of The Art of Weighing with the Clootkrans proof. Right: Statue of Stevin in Bruges showing his Clootkrans proof.

Stevin, in the presence of the mayor of Delft, dropped two unequal weights from a tower in Leiden to prove that they would reach the ground at the same time. This experiment is generally attributed to Galileo, but this was understood much earlier. In God's Philosophers: How the Medieval World Laid the Foundations of Modern Science (Hannan, 2009), it is shown how John Philiponus (490-570 CE) already knew this.

## 4. DE THIENDE

In De Thiende (1585), Stevin systematically showed how all calculations with real numbers could be reduced to standard operations with natural numbers. Stevin expands the notion of numbers from integers and fractions to "that which expresses the quantity of everything". In essence, Stevin invented the entire concept of real numbers and the number line. Any quantity could be placed on this line, including negative numbers, irrational numbers, and what he called mutually immeasurable numbers (Stepanov and Rose, 2015). In his History of Algebra Bartelt Leendert van der Waerden (1983) wrote: "Thus, with one stroke, the classical limitation of "numbers" to integers (Euclid) or to rational fractions (Diophantos) was eliminated. For Stevin, the real numbers formed a continuum. His general notion of a real number was tacitly or explicitly accepted by all
later scientists". Stepanov and Rose wrote: "His reasoning was like what eventually became known as the Intermediate Value Theorem ${ }^{1}$ which says that if a continuous function is negative at one point and positive at another, there must be an intermediate point where the value is zero. Stevin's idea was to find the interval between two consecutive integers where the function goes from negative to positive, then divide that interval into tenths, and repeat the process with the tenths, hundredths, and so on. He realized that by "zooming in," any such problem could be solved with any desired accuracy, or as he put it, "one can obtain as many decimals of the true value as one wishes and come infinitely close to it."

The importance of real numbers to science is obvious, as it inspired René Descartes to depict pairs of numbers as points in a plane. Dirk Jan Struik, editor of the mathematical volumes of Simon Stevin's Principal Works, writes: "In his arithmetical and geometrical studies, Stevin pointed out that the analogy between numbers and line segments was greater than was generally recognized. He showed that the principle of arithmetic operations, as well as the theory of ratios and the rule of three, had their counterparts in geometry. Mutual immeasurability existed between line segments and numbers.......; Mutual immeasurability was a relative property, and it made no sense to call numbers "irrational," "irregular," or by a similar name, which implied inferiority. He went so far as to say in his Traicté des incommensurable grandeurs that the geometrical theory of mutually immeasurable magnitudes, in Euclid's Tenth Book had originally been discovered in terms of numbers and translated the contents of this book into the language of numbers. He compared the still incompletely understood arithmetical continuum with the geometrical continuum already declared by the Greeks, and thus prepared the way for that correspondence of numbers and points on the line which made its appearance with Descartes' coordinate geometry."

## 5. FROM 'MULTINOMIES" TO C++

About one century after Stevin, no other than Newton used this very method in his method of fluxions. The combination of a decimal system and a positional system for describing the real numbers was formalized by Stevin, and much has been said and written about his far-reaching influence. Any polynomial - in the simplest case in one variable - can be considered as a positional system. And this was the reason for Newton's development of

[^0]infinite series. He could not understand that nobody had yet come up with the idea of using variables instead of numbers in a position system. One hundred years after the introduction of the decimal system, Newton wrote in the book in which he developed infinite series to describe change: "I keep wondering why no one ever thought of a similar application of the recently discovered Decimal Fractions to Species...., especially since they might have opened a door to deeper Discoveries...".

In essence, an equation with a left and a right side, is an application of the Law of the Lever, and the equality sign certifies that the scale is in equilibrium. As we have seen, we can play with the unit element to work in the same "dimensions". Stevin himself had also published on polynomials, or multinomials as he called them (Stevin's Principal Works, Le Livre d'Arithmetiq). Definition XXVI: Multinomie algebraique est un nombre consistent de plusieurs diverses quantitez". Explication: Comme 3(2) +5② - 4(1) +6 s'appele multinome algebraique. Et quand il aura de quantitez comme 2(1) $+4^{(2)}$ s'appelent binomie, \& de trois quantitez s'appellera trinomie, \&c."

The value of this is highlighted in the book From Mathematics to Generic Programming (Stepanov and Rose, 2015) which offers a mathematical approach for computer programmers. The first author of this fine book is Alexander A. Stepanov, a Russian mathematician who has worked in the United States since the 1980s and who is the main driving force behind the standard library of $\mathrm{C}++$, an object-oriented programming language.

Chapter 8 of this book, entitled More Algebraic Structures, deals with the historical transition from numbers to abstract objects, which formed the basis for abstract algebra and programming languages such as $\mathrm{C}++$. There are two main protagonists in this chapter, Stevin, and Emmy Noether. The latter is forever associated with the development of abstract algebra. But it already starts with Simon Stevin. Section 8.1 in (Stepanov and Rose, 2015) describes this in detail.

### 8.1. Stevin, Polynomials and Greatest common divisor

Stevin's next great achievement was the invention of polynomials with one variable, also introduced in 1585, in a book called Arithmétique. We consider the expression: $4 x^{4}+7 x^{3}-x^{2}+27 x-3$. Prior to Stevin's work, the only way to construct such a number was by using an algorithm for each polynomial. Stevin realized that a polynomial is simply a finite set of numbers: $\{4,7,-1,27,-3\}$ for the previous example.

In modern computer science terms, we might say that Stevin was the first to realize that code could be treated as data. With Stevin's insight, we can now enter polynomials as data in a generic evaluation function. We write one that uses Horner's rule, which uses the associativity of multiplication to ensure that we never need to use powers of $x$ greater than 1 :

$$
4 x^{4}+7 x^{3}-x^{2}+27 x-3=(((4 x+7) x-1) x+27) x-3
$$

For a polynomial of the $n$th degree, we need $n$ multiplications and $n-m$ additions, where $m$ is the number of coefficients equal to zero. Usually, we settle for performing $n$ additions, because checking that each addition is needed takes more steps than just performing the addition. Using this rule, we can implement an evaluation function for such a polynomial, where the arguments 'first' and 'last' indicate the limits of a set of coefficients of the polynomial.

```
template <InputIterator I, Semiring R>
    R polynomial_value (I first, I last, R x) {
        if (first == last) return R(0);
        R sum(*first)
        while (++first != last) {
        sum *= x;
        sum +x=*first;
        }
        return sum;
    }
```

Let's look at the conditions imposed on the types satisfying I and R. I is an iterator because we want to iterate over the order of the coefficients. But the value type of the iterator (the type of the coefficients of the polynomial) need not be the same as the semiring R (the type of the variable x in the polynomial). For example, with a polynomial like $a x^{2}+b$ where the coefficients are real numbers, that does not necessarily mean that $x$ itself is a real number. It could even be something completely different, such as a matrix ${ }^{2}$.

Stevin's breakthrough made it possible to treat polynomials as numbers to which normal

[^1]arithmetic operations can be applied. To add or subtract polynomials, we simply add or subtract the corresponding coefficients. To multiply, we calculate the product of each pair consisting of one coefficient of each polynomial. That is, if $a_{i}$ and $b_{i}$ are the $i$-th coefficients of the polynomials being multiplied (starting with the lowest order term), and $C_{i}$ is the $i$-th coefficient of the result, then:
\[

$$
\begin{gathered}
c_{0}=a_{0} b_{0} \\
c_{1}=a_{0} b_{1}+a_{1} b_{0} \\
c_{2}=a_{0} b_{2}+a_{1} b_{1}+a_{2} b_{0} \\
\ldots c_{k}=\sum_{k=i+j} a_{i} b_{j}
\end{gathered}
$$
\]

To divide polynomials, we need the concept of degree:
Definition 8.1. The degree of a polynomial $\operatorname{deg}(p)$ is the index of the highest coefficient different from zero (or what amounts to the same thing, the highest power of the variable)

For example

$$
\begin{gathered}
\operatorname{deg}(5)=0 \\
\operatorname{deg}(x+3)=1 \\
\operatorname{deg}\left(x^{3}+x+7\right)=3
\end{gathered}
$$

Now we can define division with remainder:
Definition 8.2. The division of a polynomial a by a polynomial b has a quotient $q$ and $a$ remainder $r$ if $q$ and $r$ are polynomials for which: $=b q+r$ and $\operatorname{deg}(r)<\operatorname{deg}(b)$. Performing division of polynomials with remainder then proceeds just like classical tail division.

Stevin understood that he could use the Euclidean algorithm to determine the greatest common divisor of two numbers to apply to polynomials as well. All that needs to be done to do this is to change the types of the objects:

```
polynomial<real> gcd(polynomial<real> a, polynomial<real>b) {
while (b != polynomial<real> (0)) {
a=remainder (a,b);
std: :swap (a,b);
}
return a;
}
```

The remainder function we use applies the algorithm for division of polynomials, although we are not concerned about the quotient. The greatest common divisor of polynomials is frequently used in computer algebra to perform such tasks as symbolic integration (Stepanov and Rose, 2015).

## 6. FINDING DOORS, LOCKS AND KEYS

Stevin's work on polynomials is generally less well known and is often considered a limited vision ("what he did not write"). But the above clearly show Stevin's deep lasting influence. The essence of generic programming is to take an efficient algorithm, generalizing it (without losing efficiency) so that it works on abstract mathematical concepts and then applying it to a variety of situations (Stepanov and Rose, 2015). Stevin's idea of using the Euclidean algorithm for polynomials exemplifies the essence of generic programming.

It is one example out of many, as the history of algebra and geometry is very old (Van der Waerden, 1983; Klein, 1992), with many main characters, before and after Stevin. Especially Emmy Noether plays a major role, the latter born almost 3,5 centuries after Stevin. Bartelt Leendert van der Waerden, author of a book about her courses, wrote: "For Emmy Noether, relationships between numbers, functions, and operations became transparent, amenable to generalization, and productive only after they were disconnected from particular objects and reduced to general conceptual relationships."

Generalities do not come through the abstract only, but real-life examples (such as lever and Clootkrans) can greatly contribute to teaching and understanding. The ideas elaborated by Stevin in a seemingly simple and didactic way have had a profound and fundamental influence on countless areas of mathematics and science (Sarton, 1934; Gielis et al., 2010). The Clootkrans proof was an important keystone. It enabled the composition of (force) vectors with the rule of the parallelogram and paved the way to
multidimensional geometry and abstract algebra. Isaac Newton was wondering why nobody thought of using variables instead of numbers, as this opens a door to deeper discoveries. Stevin already had this idea.

Simon Stevin was more than a mathematician, much more: he was a also physicist, engineer, builder (mills, fortifications, sluices), and more. From his books with its marvelous illustrations, he proves himself to be an excellent teacher. In 2020, the 400th anniversary of Simon Stevin's death was commemorated in Belgium, his home country. There was an exhibition at the Bruges City Archives of original prints of his most important works. While Stevin is highly regarded internationally, in our own country we do not recognize his profound influence on science and mathematics, if at all. He wrote in Dutch, and this is often cited as a drawback that limited Stevin's influence.

But we cannot fully appreciate his accomplishments by looking at them through only one lens. People focus more on what Stevin did not write than on what he did write (and that usually falls under the rubric of missed opportunities). But finding the key to unlock a door is much more important than opening the door bit by bit at later times. One should not forget that finding the key to unlock a door includes choosing the right door and finding the right key for it. When we look at what doors or gateways his work has opened, and when we adjust our perspective or allow for multiple perspectives, it becomes clear how deep and fundamental his impact has been on various fields. Stevin's name was (and is) known to the great mathematicians and scientists of the last four centuries.

When we talk to mathematicians and physicists of international standing, we hear a story of admiration and of very profound and far-reaching influence. Understanding Stevin requires a broader framework and a comprehensive, multifocal vision. George Sarton, a native of Ghent and one of the fathers of the history of science, himself also a mathematician, knew Stevin's works very well and wrote: "Stevin's achievements relate to the extension of the decimal idea to fractions, and to weights and measures, the theory of algebraic equations, and especially the principles of statics and hydrostatics. He was one of the greatest mathematicians of the sixteenth century and the greatest mechanic of the long period that stretched from Archimedes to Galileo".

But also: "Taking Galilei out of category, Stevin was undoubtedly the most original man of the second half of the sixteenth century, but he has not yet received the full fame he deserves. This may seem strange, for his greatness is remarkable and not only in one domain, but in many. On the other hand, it can be argued that he is less well known
because of his originality and that exactly that therefore becomes a kind of confirmation of his genius. And how could people really admire someone they don't understand, how could they regard a great man whose greatness they have not yet learned to appreciate? (Sarton, 1934)".

## REFERENCES

Bosmans H. (1923) Le calcul infinitesimal chez, Simon Stevin, Mathesis, 12-19.
Bosmans H. (1926) Le mathematicien belge, Simon Stevin de Bruges (1548-1620), Periodico di Mathematice Serie IV, N.4: 231-261.
Devreese J.T., Vanden Berghe G. (2008) 'Magic is No Magic' The wonderful world of Simon Stevin, WIT Press, Southampton.
Feynman R., Leighton R.B., Sands M. (1963) The Feynman Lectures on Physics, Addison-Westley, https://doi.org/10.1063/1.3051743.
Gielis, J., Caratelli, D., Haesen, S., \& Ricci, P. E. (2010) Rational mechanics and science rationelle unique, In The Genius of Archimedes - 23 Centuries of Influence on Mathematics, Science and Engineering, (pp. 29-43), Springer, Dordrecht, https://doi.org/10.1007/978-90-481-9091-1_3.
Gielis, J., Verhulst, R., Caratelli, D., Tavkelidze, I., Ricci, P.E. (2014) On means, polynomials, and Special Functions, Teaching of Mathematics, 17(1): 1-20.
Hannam J. (2009) God's Philosophers: How the Medieval World Laid the Foundations of Modern Science. Icon Books, London.
Klein J. (1992) Greek mathematical thought and the origin of algebra, Dover Publications.
Mach, E. (1893) The science of mechanics: A critical and historical exposition of its principles, Open court publishing Company.
Pickover C.A. (2008) Archimedes to Hawking: laws of science and the great minds behind them. Oxford University Press Inc., New York.
Sarton, G. (1934) Simon Stevin of Bruges (1548-1620), ISIS, 21(2): 241-303, https://doi.org/10.1086/346851.
Sarton, G (1935) The First Explanation of Decimal Fractions and Measures (1585). Together with a History of the Decimal Idea and a Facsimile (No. XVII) of Stevin's Disme, Isis, 23(1): 153-244, https://doi.org/10.1086/346940.
Stevin, S (1955) Principal Works, C.V. Swets and Zeitlinger.
Stepanov, A.A., Rose, D.E. (2015) From Mathematics to Generic Programming, Addison-Wesley, Boston.
Van der Waerden B.L. (1983) A History of Algebra. From al-Khwarizmi to Emmy Noether, Springer.
Verstraelen L. (2014) A concise mini history of geometry, Kragujevac Journal of Mathematics, 38(1):5-21, https://doi.org/10.5937/KgJMath1401005V.


[^0]:    ${ }^{1}$ In Stepanov \& Rose (2015) this is described in the Section "Origins of Binary Search."

[^1]:    ${ }^{2}$ Iterators are introduced formally in Chapter 10 of Stepanov \& Rose (2015)

