EUROPHYSICS LETTERS 1 May 2006

Europhys. Lett., **74** (3), pp. 493–499 (2006)
DOI: 10.1209/epl/i2006-10013-1

Superconducting films with antidot arrays —Novel behavior of the critical current

G. R. Berdiyorov, M. V. Milošević and F. M. Peeters

Departement Fysica, Universiteit Antwerpen
Groenenborgerlaan 171, B-2020 Antwerpen, Belgium

received 16 January 2006; accepted in final form 3 March 2006
published online 29 March 2006

PACS. 74.25.Sv – Critical currents.
PACS. 74.78.-w – Superconducting films and low-dimensional structures.
PACS. 74.25.-q – Properties of type I and type II superconductors.

Abstract. – Novel behavior of the critical current density j_c of a regularly perforated superconducting film is found, as a function of applied magnetic field H. Previously pronounced peaks of j_c at matching fields were always found to decrease with increasing H. Here we found a *reversal of this behavior* for particular geometrical parameters of the antidot lattice and/or temperature. This new phenomenon is due to a strong "caging" of interstitial vortices between the pinned ones. We show that this vortex-vortex interaction can be further tailored by an appropriate choice of the superconducting material, described by the Ginzburg-Landau parameter κ. In effective type-I samples we predict that the peaks in $j_c(H)$ at the matching fields are transformed into a *step-like behavior*.

Introduction. – For practical applications of superconducting (SC) materials, the increase and, more generally, control of the critical current in SC samples are of great importance. In recent years, much attention was given to the investigation of superconducting films patterned with a regular array of microholes (antidots), which have a profound influence on both the critical current and the critical magnetic field [1–5]. Due to the collective pinning to the regular antidot array, vortices are forced to form rigid lattices when their number "matches" integer and fractional multiples of the number of pinning sites at fields $H_n = n\Phi_0/S$, and $H_{p/q} = \frac{p}{q}\Phi_0/S$ (where n, p, q are integers) respectively, where Φ_0 is the flux quantum, and S is the area of the unit cell of the antidot lattice. This locking between the pinning array and the vortex lattice is responsible for the reduced mobility of the vortices in applied drive and consequently the increased critical current at integer and fractional matching fields, was confirmed both by experiments (imaging [6], magnetization and transport measurements [1,2]) and molecular-dynamics simulations [7].

However, regardless of the imposed pinning profile, the vortices at interstitial sites always have high mobility [5, 8], show different dynamic regimes from the pinned ones, and their appearance is followed by a sharp drop in the critical current [1]. In this respect, the maximal occupation number of the antidots (saturation number n_s) becomes very important for any study of the critical current. In an early theoretical work, Mkrtchyan and Schmidt [9] have shown that n_s depends only on the size of the holes. However, in the case of periodic pinning arrays this number depends also on the proximity of the holes, on temperature and on the applied field [1, 5, 10, 11].

Besides the pinning strength of the artificial lattice, vortex-vortex interactions are crucial for vortex dynamics. Most of the experiments on perforated superconducting films are carried out in the effective type-II regime ($\kappa_* = 2\kappa^2/d \gg 1/\sqrt{2}$, with κ being the Ginzburg-Landau (GL) parameter, and d being the thickness of the SC film scaled to the coherence length ξ), where vortices act like charged point-particles, and their interaction with the periodic pinning potential can be described using molecular-dynamics simulations (MDS) [7]. However, the overlap of vortex cores (with sizes $\sim \xi$), and the exact shape of the inter-vortex interaction (depending on the material properties reflected through κ), which are neglected in MDS, may significantly modify the equilibrium vortex structures and consequently the critical current.

In the present letter we investigate the critical current j_c of superconducting films with regular arrays of square antidots, taking into account all parameters relevant to the SC state, within the non-linear Ginzburg-Landau theory. This formalism allows us to analyze the j_c dependence on the geometrical parameters of the sample, material (even type-I) and temperature, and compare our results with existing experiments.

Theoretical approach. – We consider a thin superconducting film (of thickness d) with a regular lattice of square holes (side a, period W) immersed in an insulating medium with a perpendicular uniform applied field H (see the inset of fig. 1). To describe this system, we solved the nonlinear Ginzburg-Landau (GL) equations for the order parameter ψ and the vector potential $\vec{A}$ (in dimensionless units, and with temperature T taken explicitly in units of the zero-field critical temperature T_{c0}, see ref. [12] for more details):

$$\left(-i\vec{\nabla} - \vec{A}\right)^2 \psi = \psi\left(1 - T - |\psi|^2\right), \tag{1}$$

$$-\kappa_* \Delta \vec{A} = \frac{1}{i}\left(\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - |\psi|^2 \vec{A}. \tag{2}$$

The magnitude of the applied field H is determined by the number of flux quanta piercing through the simulation region $W_s \times W_s$. On the hole-edges we used boundary conditions corresponding to zero normal component of the superconducting current. Periodic boundary conditions [13] are used around the square simulation region: $\vec{A}(\vec{r} + \vec{l}_i) = \vec{A}(\vec{r}) + \vec{\nabla}\zeta_i(\vec{r})$ and $\psi(\vec{r} + \vec{l}_i) = \psi \exp[2\pi i \zeta_i(\vec{r})/\Phi_0]$, where $\vec{l}_i$ ($i = x, y$) are lattice vectors and ζ_i is the gauge potential. We use the Landau gauge $\vec{A}_0 = Hx\vec{e}_y$ for the external vector potential and $\zeta_x = HW_s y + C_x$, $\zeta_y = C_y$, with C_x, C_y, being constants. Generally speaking, depending on the geometry of the antidot lattice, one must minimize the energy with respect to the latter coefficients. The size of the supercell in our calculation is typically 4×4 unit cells (containing 16 holes, *i.e.* $W_s = 4W$). We solved the system of equations (1)-(2) self-consistently using the numerical technique of ref. [12]. In order to calculate the critical current, first we determine the ground vortex-state for a given applied magnetic field after multiple starts from a randomly generated initial Cooper-pair distribution. Then the applied current in the x-direction is simulated by adding a constant A_{cx} to the vector potential of the applied external field [14]. Note that the current j_x in the sample resulting from the applied A_{cx} is obtained after integration of the x-component of the induced supercurrents (calculated from eq. (2)) over the y = const cross-section of the sample. With increasing A_{cx}, the critical current is reached when a stationary solution for eqs. (1)-(2) ceases to exist (*i.e.* vortices are driven in motion by the Lorentz force).

Influence of geometrical parameters of the sample. – The enforced stability of the vortex lattices in periodically perforated superconducting samples [6, 15] at integer and fractional matching fields leads to pronounced peaks in the critical current [1–3]. However, the exact shape of these lattices, and consequently their stability when locked to the pinning arrays, strongly depend on the parameters of the sample. For example, while small antidots can pin only one vortex, in larger holes multi-quanta vortices may become energetically preferable [10]. This reduces the number of interstitial vortices, whose higher mobility affects strongly the critical current of the sample. Therefore, in this section we investigate the critical current of our sample for different sizes of the antidots a and antidot lattice period W.

Figure 1 shows the critical current density j_c of the sample as a function of the applied magnetic field H for three different values of the antidot size: $a = 1.5\xi$, $a = 2.6\xi$ and $a = 4\xi$. The lattice period is $W = 8\xi$, the film thickness $d = 0.1\xi$ and GL parameter equals $\kappa = 1$ (corresponding roughly to Pb, Nb, or Al films). Note that, following the suggestion of Wahl [16], we took into account the influence of the perforation on the effective GL parameter $\kappa_* = 2\kappa^2/d(1 - 2a^2/W^2)$. As shown in fig. 1, in the absence of applied field the samples with smaller antidots always have larger j_c simply due to more superconducting material, *i.e.* larger screening. However, for $H \neq 0$, the critical current depends on the vortex structure in the sample. For small antidot-size $a = 1.5\xi$, where only one vortex can be captured by each hole (see the inset of fig. 1), the $j_c(H)$ curve shows the expected maxima at integer matching fields H_1, H_2, H_3 and H_4 and at some of the fractional matching fields. As observed before [1], the peaks of the critical current at matching fields decrease with increasing magnetic field. However, a novel phenomenon is found for $2.5 \leq a/\xi \leq 2.8$: *the critical current at the 3rd matching field is larger than the one at the 2nd matching field.* Moreover, the critical currents for $H_2 < H < H_3$ are higher than those obtained for fields $H_1 < H < H_2$.

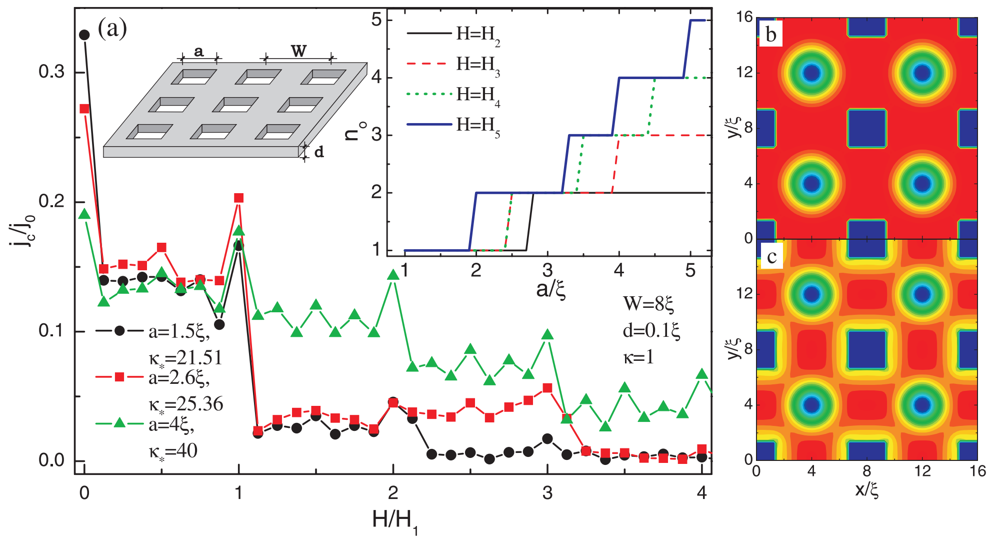

Fig. 1 – (Color online) The critical current density j_c (a) (in units of $j_0 = cH_{c2}\xi/4\pi\lambda^2$) as a function of the applied magnetic field H (in units of the first matching field H_1) for different antidot sizes and fixed period $W = 8\xi$, and the contour plot of the Cooper-pair density at $H = H_2$ (b) and at $H = H_3$ (c) for $R = 2.6\xi$. The insets show the schematic view of the sample and the antidot occupation number n_o as a function of the antidot size a at different matching fields (*i.e.* for 2–5 vortices per unit cell).

The explanation for this counterintuitive feature of j_c lies in the hole occupation number n_o, *i.e.* the number of vortices inside the hole, and, consequently, in the saturation number n_s of the holes ($n_s = n_o$ for larger fields). For holes with $2.5 \leq a \leq 2.8$, n_s is equal to 1 ($n_s \approx a/4\xi$ [9]). Therefore, at $H = H_1$, all vortices are captured by the antidots. Analogously, at $H = H_2$, besides the pinned vortices, one vortex occupies each interstitial site. If the same analogy is followed further, one expects two vortices at each pinning site at the 3rd matching field. However, this depends also on the distance between neighboring holes. Namely, the period W determines the distance between the interstitial vortices. In other words, for small W, this distance is small, vortex-vortex repulsion is large, and it may become energetically more favorable for the system to force one more flux-line into the hole rather than have two vortices at interstitial sides (see figs. 1(b), (c)). Therefore, our results show that the occupation number n_o of the holes in a lattice *depends not only on the size of the hole a, but also on the period W and the number of vortices per unit-cell of the hole lattice*. For the parameters given above, $n_o = 2$ at $H = H_3$, *i.e.* only one vortex is located at each interstitial site, as for the case of $H = H_2$. Note that this interstitial vortex interacts repulsively with the pinned ones. This interaction is roughly twice as large at $H = H_3$ than at $H = H_2$, and the interstitial vortex becomes effectively "caged". This "caging" effect has been found for $7\xi \leq W \leq 8.3\xi$ for the radius $R = 2.6\xi$. For $W > 8.3\xi$, n_o and n_s become independent of W and for $W < 7\xi$, the larger suppression of superconductivity around the holes at $H = H_3$ becomes more significant than "caging", and j_c reverses again in favor of $H = H_2$.

With further increase of hole size a, more vortices are captured by the holes. Consequently, in order to observe the "caging" effect, one needs to consider higher magnetic fields, *i.e.* for $n_o = 4$, at least $H > H_5$ is needed. In any case, this effect can always be realized at a given magnetic field by an adequate choice of a and W. Figure 2 shows the critical current density of our sample for different periods of the antidot lattice W at $a = 2.6\xi$. For clarity, we plotted only the critical current at the matching fields. At $H = 0$ and $H = H_1$ (when there are no interstitial vortices) the critical current is an increasing function of W, because of the stronger screening by larger quantities of the superconducting material. For higher magnetic fields interstitial vortices nucleate more easily in a sample with larger W, which leads to a

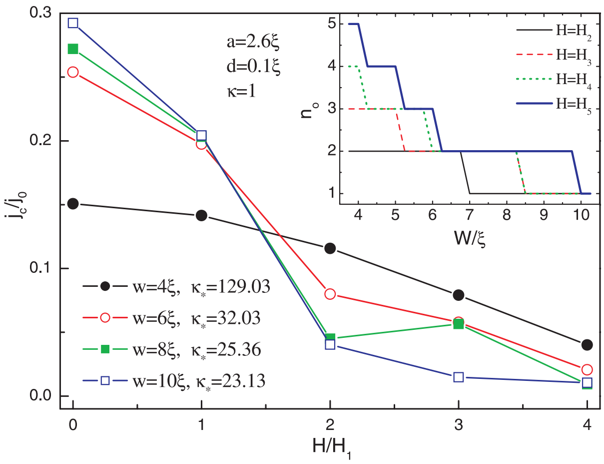

Fig. 2 – (Color online) The critical current density j_c of the superconducting film at matching fields for different periods of the antidot lattice. The inset shows the antidot occupation number n_o as a function of W at different matching fields.

reduction of j_c. The dependence of n_o (and thus also saturation number n_s) on W is shown in the inset of fig. 2: n_o decreases from $n_o = 5$ to $n_o = 1$ at $H = H_5$, when we increase W from 4ξ to 10ξ.

Temperature dependence of the critical current. – So far, we presented results of our calculations at fixed temperature, where all units were temperature dependent (*e.g.* distances were expressed in $\xi(T)$). In what follows, we consider a superconducting film with thickness $d = 20\,\text{nm}$, interhole distance $W = 1\,\mu\text{m}$, and antidot size $a = 0.33\,\mu\text{m}$. We choose the coherence length $\xi(0) = 40\,\text{nm}$ and the penetration depth $\lambda(0) = 42\,\text{nm}$, which are typical values for Pb films. We study the influence of temperature with the help of the right-side term of eq. (1), which actually describes the temperature dependence of the coherence length $\xi(T) = \xi(0)/\sqrt{|1 - T/T_{c0}|}$ and penetration depth $\lambda(T) = \lambda(0)/\sqrt{|1 - T/T_{c0}|}$.

Figure 3 shows the calculated critical current density of the sample as a function of the applied field normalized to the first matching field at temperatures $T/T_{c0} = 0.86$, 0.88, 0.9, 0.92, 0.94, 0.96 and 0.98. As discussed in the previous section, the $j_c(H)$ curve shows pronounced maxima at matching fields with a substantial drop after the number of vortices per unit-cell exceeds the hole-saturation number (in our case, for $H > H_1$).

As expected, decreasing temperature results in an increase of the critical current for given magnetic field. However, the qualitative behavior of the $j_c(H)$ characteristics changes. Although the W/a ratio remains the same, the size of vortices and the occupation number of the holes may change with temperature (as $\xi(T)$ changes). Actually, we found the same vortex structure at all temperatures $T > 0.8T_{c0}$, but $j_c(H_3)$ is found to be larger than $j_c(H_2)$ only in the temperatures range $0.8T_{c0} \leq T \leq 0.93T_{c0}$. At higher temperatures vortices become larger and suppression of superconductivity around the holes "masks" the critical current enhancement by the "caging" effect. This decrease of Cooper-pair density around the holes is also responsible for the decreased $j_c(H_1)/j_c(0)$ ratio with increasing temperature. At temperatures lower than $0.8T_{c0}$, the peaks of j_c again decrease with magnetic field, since W/ξ significantly increases and its influence on n_o diminishes (see the previous section).

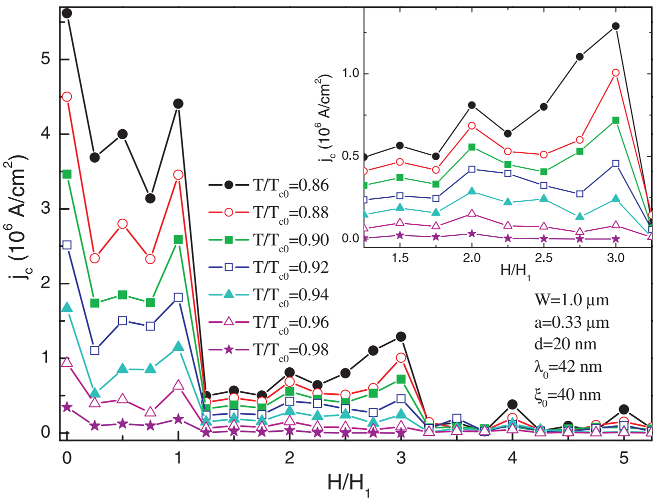

Fig. 3 – (Color online) The temperature dependence of $j_c(H)$ for a superconducting film containing an antidot array, at temperatures $T/T_{c0} = 0.86$–0.98. The period of the antidot lattice is $W = 1\,\mu\text{m}$, the antidot size $a = 0.33\,\mu\text{m}$, film thickness $d = 20\,\text{nm}$, and κ equals 1.05.

Interestingly enough, this $j_c(H_{n+1})$ *vs.* $j_c(H_n)$ reversing behavior *was recently found experimentally, but not noticed*. In fig. 6(b) of ref. [4] a clear enhancement of the critical current at $H/H_1 = 3$ was found which is larger than the one at H/H_2 as is similar to the behavior found in fig. 3 for $T/T_{c0} < 0.94$. A quantitative comparison between theory and experiment is difficult because of the different determination of j_c. In our calculations we assume normal state as soon as vortices are set in motion, whereas in transport measurements a certain value of threshold voltage determines the critical current.

Effective type-I vs. type-II behavior of the critical current. – In the previous sections we have shown that higher mobility of interstitial vortices leads to a dramatic decrease of the critical current. Therefore, keeping the interstitial sites of the superconductor vortex-free is essential for an improvement of j_c. In this respect, we consider effectively type-I superconductors, where i) the screening of the magnetic field is enhanced (*i.e.* vortices will be compressed in the holes), and ii) the interaction of vortices becomes attractive (depends on $\ln \kappa_*$). As an example we considered a sample, with antidot size $a = 0.5\,\mu$m, lattice period $W = 1.5\,\mu$m, $\xi_0 = 40$ nm and $\lambda_0 = 10$ nm. We fine-tuned the effective vortex-vortex interaction by varying the thickness of the sample, *i.e.* changing from type-II to type-I behavior with increasing d. Figure 4 shows the critical current of the sample for four values of the film thickness: $d = 10$ nm (solid dots), $d = 50$ nm (open dots), $d = 100$ nm (solid squares) and $d = 150$ nm (open squares) at $T = 0.97T_{c0}$. For $d = 10$ nm, the sample is still in the type-II regime ($\kappa_* = 2.887$), and the critical current shows a peak-like behavior at the matching fields. The drop in the critical current for $H > H_1$ is caused by the interstitial vortices which is larger with increasing κ_*. Note that the "caging" effect is also present for these values of the parameters. When the film thickness is increased ($d = 50$ nm, $\kappa_* = 0.577$) the critical current density is higher for $H < H_2$. For $H < H_1$, this increase is achieved due to a stronger Meissner effect. At fields $H_1 < H < H_2$ the increase of j_c is more apparent, as all vortices are captured by the holes (except for $d = 10$ nm, where $n_o = 1$). As soon as interstitial vortices appear in the sample ($H > H_2$), j_c becomes even smaller than the one for $d = 10$ nm. This inversion of j_c clearly demonstrates the higher mobility of interstitial vortices in type-I superconductors.

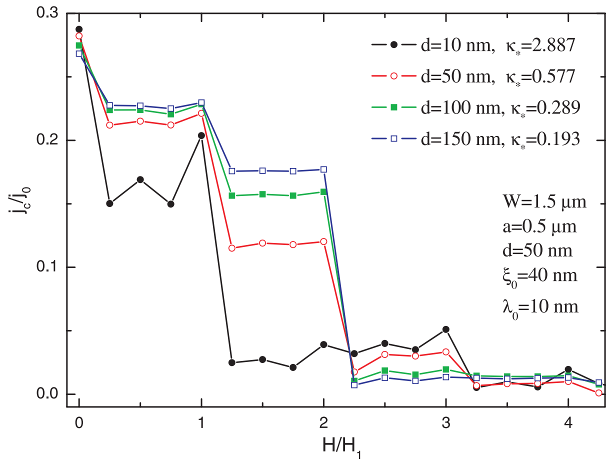

Fig. 4 – (Color online) The critical current density j_c as a function of the applied magnetic field for different thickness of the sample. Remaining parameters are listed in the figure.

Looking at the $j_c(H)$ curve as a whole for $d = 50\,\mathrm{nm}$, we observed a pronounced *step-like behavior*. Note that in type-I samples the matching between the number of flux-lines and the number of antidots does not lead to a peak-like increase of the critical current. Namely, regardless of their number, additional flux lines are doubly pinned by the attractive hole-potential and the attractive interaction with previously pinned vortices. The "step" in $j_c(H)$ occurs only when the number of vortices per hole n_o changes (more vortices and consequently a larger suppression of ψ around the hole), or when interstitial vortices appear. The effect of an increase of n_o diminishes as d increases (*i.e.* κ_* decreases and screening increases), or when temperature is lowered (as discussed before). This tendency is illustrated by solid and open dots in fig. 4, and ultimately leads to a *two-step $j_c(H)$ curve*, with larger critical current for $H < H_n$, and smaller j_c for $H > H_n$, where $n = n_o$.

Conclusions. – We studied the critical current of a superconducting film containing an array of antidots in a uniform magnetic field, as a function of all relevant parameters. We found that the well-known j_c enhancement by artificial vortex pinning strongly depends on the antidot occupation number n_o. The latter is determined not only by the size of the antidots as commonly believed, but *also by their spacing and the applied field*. As a consequence, when the parameter conditions are met, the critical current becomes larger *at higher matching fields*, contrary to the conventional behavior. Such a feature is a result of the "caging" of interstitial vortices between the larger number of pinned ones. This effect is strongly influenced by temperature, which agrees with a recent experiment. [4] Additionally, the interactions in this system can be tailored by the effective Ginzburg-Landau parameter κ_*. In effectively type-I samples, vortices attract, j_c increases and peaks at matching fields diminish due to the enforced pinning at all fields. As a result, a *novel step-like behavior* of the critical current is found, with a sharp drop at higher fields, when interstitial vortices appear.

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This work was supported by the Flemish Science Foundation (FWO-Vl), the Belgian Science Policy (IUAP), the JSPS-ESF NES program, and the ESF-AQDJJ network.

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