

Current-Voltage Characteristics of Quasi-One-Dimensional Superconductors: An *S*-Shaped Curve in the Constant Voltage Regime

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Applying a constant voltage to superconducting nanowires we find that its I-V characteristic exhibits an unusual *S* behavior. This behavior is the direct consequence of the dynamics of the superconducting condensate and of the existence of two different critical currents: j_{c2} at which the pure superconducting state becomes unstable and $j_{c1} < j_{c2}$ at which the phase-slip state is realized in the system.

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The majority of the experiments on the resistive state in quasi-one-dimensional systems were performed in the constant current regime and at temperatures close to T_c . It is extremely difficult to apply voltage to a superconductor because the current density induced by the applied electric field inevitably reaches the critical value and destroys superconductivity in the sample. The decrease of the superconducting current by the appearance of phase-slip centers [1–5] is not effective in this case because of the large heating of the sample at low temperatures. At temperatures close to T_c the heating can be suppressed due to the low value of the critical currents, but in this case the applied electric field does not penetrate deep into the sample because of the existence of regions near the normal-superconductor boundary where the drop of the applied voltage occurs [6,7].

This situation drastically changes with the appearance of nanotechnology and the ability to create long (to allow the appearance of phase-slip centers) superconducting wires with a small cross section (to decrease the effect of heating). In this Letter we present results on the behavior of such nanowires in the constant voltage regime. We found that the I-V characteristic in this case has a remarkable *S* shape. Our theoretical analysis based on the time-dependent Ginzburg-Landau equations (TDGL) shows that such a behavior is a direct consequence of the dynamics of the superconducting condensate, and we predict new unusual features which still need additional experimental study.

The superconducting nanowires were prepared by electrodeposition into nanopores of homemade track-etched polycarbonate membranes [8]. For the lead nanowires, a 22 μm thick membrane (with pore diameter ~ 40 nm and pore density $\sim 4 \times 10^9 \text{ cm}^{-2}$) and an aqueous solution of 40.4 g/l $\text{Pb}(\text{BF}_4)_2$, 33.6 g/l HBF_4 , and 15 g/l H_3BO_3 were used [9], while in the case of the tin nanowires, a 50 μm thick membrane (with pore diameter ~ 55 nm and pore density $\sim 2 \times 10^9 \text{ cm}^{-2}$) and an electrolyte of 41.8 g/l $\text{Sn}(\text{BF}_4)_2$ in water solution were applied. The constant potential of -0.5 V versus an Ag/AgCl refer-

ence electrode was used in a three-electrode configuration in order to reduce the Pb^{2+} or Sn^{2+} ions into the nanopores. As shown in Fig. 1, the nanowires are cylindrical and the diameter is uniform along their length. In order to perform electrical transport measurements on a single nanowire (still kept inside the membrane), a self-contacting technique was developed. In addition to the thick gold cathode ($\sim 1 \mu\text{m}$) from which the nanowires start to grow up inside the membrane, another thin gold layer (in the range of 50 to 200 nm) was deposited on the other side exposed to the electrolyte prior to the electrodeposition. This contacting layer covers the surface but in contrast to the cathode, it is not thick enough to plug the pores. As the nanowires do not grow at the same speed, the first emerging nanowire interrupts the growth of the others by favoring the formation of a film of the deposited material on this contacting layer.

All the transport measurements are done in an electromagnetically shielded environment and special



FIG. 1. Scanning electron microscopy–field emission gun micrograph of lead nanowires after dissolution of the hosting polycarbonate membrane with dichloromethane.

precautions were taken to reduce the noise. The voltage across the sample was measured by a Keithley voltmeter. From the numerous measurements performed both on Pb and on Sn nanowires, it appears that almost no depression of T_c was observed. In addition, a superconducting transition was found to occur when the residual resistivity ratio (RRR) is larger than about 2, while for smaller RRR values an insulating behavior is observed [9]. Our nanowires are moderately disordered and the RRR of the samples was larger than 10. Structural characterization of the Pb nanowires using TEM experiments confirm also their polycrystalline character [10].

In Fig. 2, the dc current-voltage (I-V) characteristics of the Pb and Sn nanowires are reported, applying either a dc current or a dc voltage. All the curves presented in Fig. 2 were measured in both directions and no hysteresis was observed. These measurements were also performed with reversed polarity without any changes compared to the reported results. In the usual current driven experiment we observe for the Pb nanowire [Fig. 2(a)] the successive appearance of two phase-slip centers (PSCs) at low temperature. Applying the phenomenological Skocpol-Beasley-Tinkham model [1], the size of the first

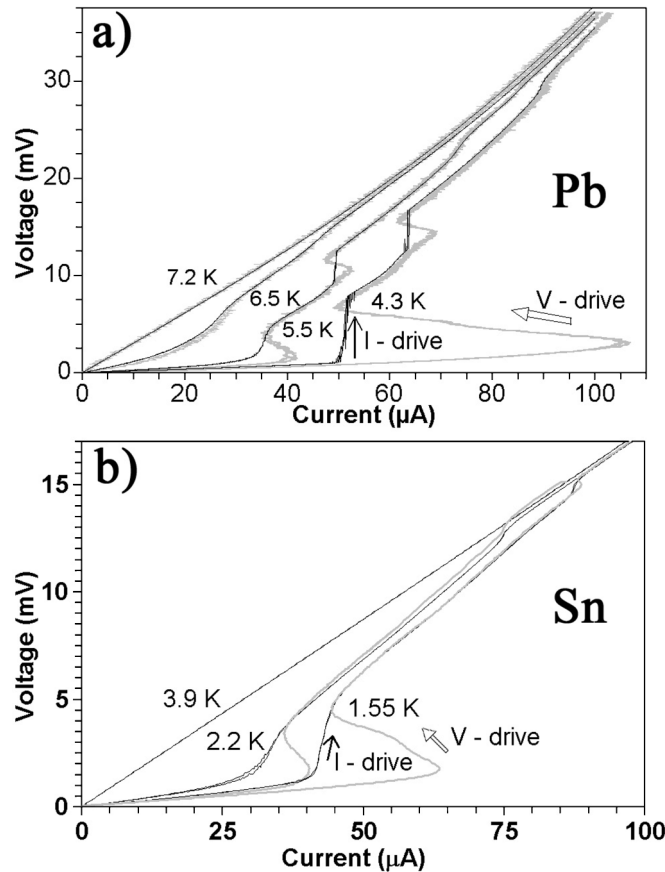


FIG. 2. Current-voltage characteristics at different temperatures of (a) a Pb nanowire (diameter 40 nm, length 22 μm) and (b) a Sn nanowire (diameter 55 nm, length 50 μm). Results are shown for the current driven mode (black curves) and the voltage driven mode (grey curves).

PSC can be estimated to be about 18 μm , which is twice the quasiparticles diffusion length. In spite of the fact that successive PCSs tend to avoid those already in place, the second PSC created in this Pb nanowire is thus forced to interpenetrate the first one, which explains why the second jump in resistance is smaller. The current driven experiment on the 50 μm long Sn nanowire [Fig. 2(b)] also shows the formation of a PSC at low temperature but for this particular material, the PSC extension is larger (around 40 μm) and the formation of a second PSC almost coincides with the transition to the normal state. Interesting new features were observed when measuring the reverse way, i.e., applying constant voltage and measuring the current. Here, the current flowing into the nanowire was determined by measuring the voltage across a 1 Ω resistance added in series and the voltage developed across the sample was measured separately. In this voltage driven experiment, we found an interesting S-shape behavior which occurs at low temperature in the formation region of these PSCs, both for the Pb and the Sn nanowires.

To understand the experimental results we used the generalized TDGL equation [2] which describes the dynamics of the superconducting condensate $\psi = |\psi|e^{i\theta}$,

$$\frac{u}{\sqrt{1 + \gamma^2 |\psi|^2}} \left(\frac{\partial}{\partial t} + i\varphi + \frac{\gamma^2}{2} \frac{\partial |\psi|^2}{\partial t} \right) \psi = \frac{\partial^2 \psi}{\partial s^2} + \psi(1 - |\psi|^2). \quad (1)$$

The physical quantities are presented in dimensionless units (see, e.g., Ref. [3]), and we neglected the vector potential because, in our case, the self-induced magnetic field is negligible in size. Equation (1) is supplemented with an equation for the electrostatic potential φ which is obtained from the condition for the conservation of the total current in the wire, i.e., $\text{div} \mathbf{j} = 0$. The parameter γ depends on the material, and we took $u \approx 5.79$ [2].

Although Eq. (1) is valid only close to T_c (see [2]) we expect to find clues of the dynamics of the system even far from T_c . This expectation is motivated by the fact that results found from the solution of the stationary Ginzburg-Landau equations for mesoscopic superconductors [11] agree with experiment even far from T_c .

If we apply an external current to the wire, then the system exhibits hysteretic behavior [2,4]. If one starts from the superconducting state and increases the current the superconducting state switches to the resistive superconducting or normal state at the upper critical current j_{c2} . With decreasing current it is possible to keep the sample in this state even for currents up to $j_{c1} < j_{c2}$ (which we call the lower critical current). The superconducting resistive state is realized as a periodic oscillation of the order parameter in time at one point of the superconductor [2,3]. When the order parameter reaches zero in this point, a phase slip of 2π occurs. Such a state is now called a phase-slip state and the corresponding point a phase-slip center. The meaning of the current j_{c1} is that

for $j < j_{c1}$ the phase-slip solution cannot be realized (in the absence of fluctuations) and thus the current j_{c1} is the critical current at which PSCs start to appear. Thus there is a limiting cycle (see Ref. [12]) for Eq. (1) giving rise to this process.

The existence of such two critical currents is, in the $V = \text{const}$ regime, responsible for a complicated I-V characteristic (see Fig. 3). The I-V curve has an overall S-like behavior which at low voltages exhibits an oscillatory behavior superimposed on it. We found that these unusual properties are typical for superconducting wires with $\gamma \gg 1$. To understand the physics leading to this unusual behavior we consider the dynamics of ψ in the wire for different voltages.

When we apply a voltage V to the wire of length L a constant electrical field appears in the sample $E = j_E = V/L$. As a result the superconducting condensate will be accelerated and the momentum $p = \nabla\theta$ will increase in time. For small voltages and electric fields the absolute value of the order parameter and current density are approximately described by quasiequilibrium expressions with $|\psi|(t) = 1 - p(t)^2$ and $j_s(t) = p(t)[1 - p(t)^2]$ [$p(t) \approx j_E t$] while $p < p_c$ (at $L/\xi \gg 1$ we have $p_c \approx 1/\sqrt{3}$). For $p > p_c$ the above spatial independent solution becomes “unstable” (as for the case of a ring in the presence of an applied magnetic field as discussed in Refs. [13,14]). The order parameter vanishes in the center of the wire and a jump of the phase by 2π occurs in this point. It decreases p by $\Delta p = 2\pi/L$ (and hence the superconducting current density) in the entire wire. A second phase slip will appear in the same point and the phase-slip process repeats itself as long as $j > j_{c1}$. When the current density becomes less than j_{c1} the order parameter will increase till its quasiequilibrium value $|\psi|(t) = 1 - p(t)^2$ with $p(t) < p_c$. Then the applied electric field will again accelerate the superconducting condensate and the above process is repeated (see Fig. 4). We define the time which is necessary to decrease the order

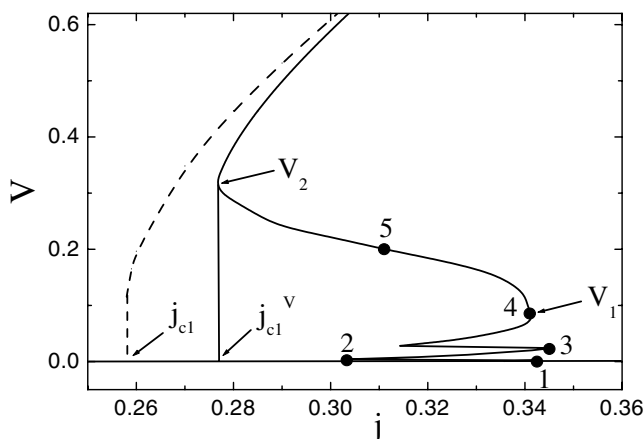


FIG. 3. The current voltage characteristics of a superconducting wire of length $L = 40\xi$ and $\gamma = 10$ ($\lambda_Q \approx 2.3\xi$). Dotted (solid) curve for the I-V = const regime.

parameter from $|\psi| = \sqrt{2/3}$ till zero and again back to the value $|\psi| = \sqrt{2/3}$ as the transition time T_{tr} and call this entire process the *transition process*.

It is easy to show that at small voltages (in the limit $V \rightarrow 0$) the number of phase slips which occurs during the transition process in an ideal wire is given by the following expression: $N_{\min} = N_{\text{int}}[(p_c - p_{c1})(L/2\pi + 1)]$, where p_{c1} is the smallest real root of equation $j_{c1} = p_{c1}(1 - p_{c1}^2)$ and $N_{\text{int}}(x)$ returns the nearest integer value. The number of phase slips occurring during the transition process increases with increasing wire length.

Because of the presence of an external electric field, p also increases during the transition process. Roughly speaking, if the voltage in the wire is so large that $j_E T_{tr} > 2\pi/L$ the number of phase slips during the transition will increase by one (see Fig. 4). It leads to an instant decrease of the average current. The reason is that the maximum current during current oscillations j_{\max} for a small change of V does not change but the minimum value of the current j_{\min} decreases by about $2\pi/L$, and hence $\langle I \rangle$ should decrease. This effect is most pronounced in short wires where N_{\min} is small and the term $2\pi/L$ is large. If N does not change with increasing V , then $\langle I \rangle$ increases because both minimum and maximum currents grow.

When the period $T = 2\pi N/V$ becomes of order $2T_{tr}$ the dynamics of the system changes considerably (at voltage V_1 in Fig. 3). Starting from this point the average current $\langle I \rangle$ monotonically decreases with increasing applied voltage. For these values of the voltages the system is unable to return to the “quasiequilibrium” state (it is a state such that if we switch off the voltage instantly the system remains in this state for an infinitely long time in the absence of fluctuations). At any moment of time the order parameter is more suppressed in the point where the oscillations occur. At these voltages j_{\max} decreases with increasing V and in principle it should reach the current

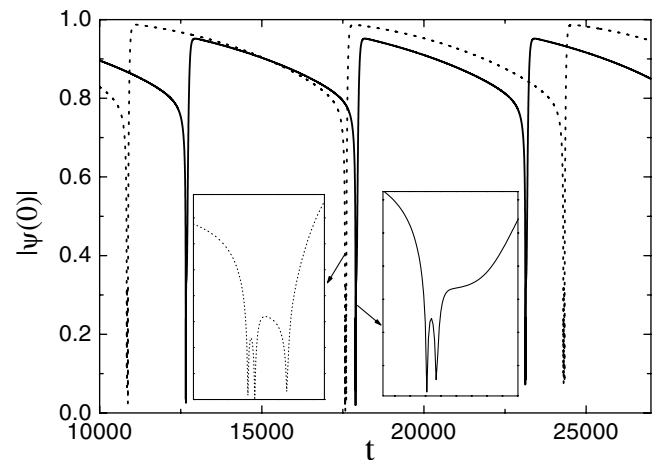


FIG. 4. Dependence of the order parameter in the center of the wire (in the minimal point of $|\psi(s)|$) as a function of time. The solid (dotted) curve corresponds to the voltage in point 1 (2) [$V = 0.0024$ ($V = 0.0028$)] of Fig. 3.

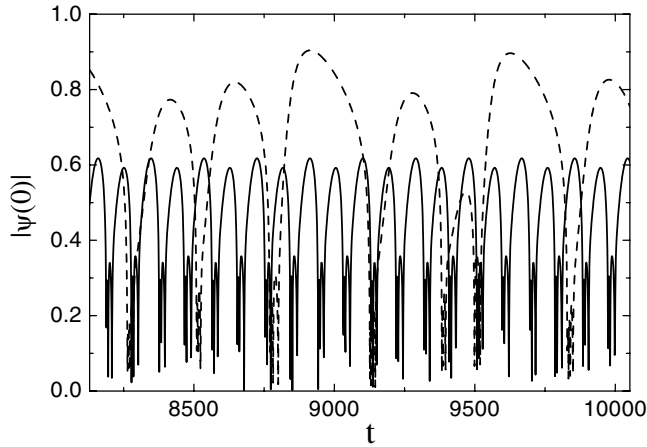


FIG. 5. Time dependence of the order parameter in the current decreasing regime for a long wire ($L = 40\xi$). Dotted (solid) curve corresponds to the voltage in point 4 (5): $V = 0.08$ ($V = 0.2$) of Fig. 3.

j_{c1} (and j_{\min} should reach j_{c1} from the bottom). But this is possible only in an infinitely long wire. In a finite length wire every phase-slip event leads to a decrease of the momentum p and the current j by a finite value. As a result the lower critical current in the $V = \text{const}$ regime is equal to $j_{c1}^V = j_{c1} + dj$ (with $dj \sim 1/L$) at which the “usual,” as in the $I = \text{const}$ regime, periodic phase-slip processes with period $T = 2\pi/\langle V \rangle$ starts. This corresponds to the voltage V_2 in Fig. 3.

In long wires a complex behavior is found in the current decreasing region due to the existence of several phase-slip events during the transition process. At the initial parts of this region an irregular behavior is typical. In Fig. 5 we show (dotted curve) an example of such a situation (at $V = 0.08$ —point 4 in Fig. 3). Not a single-period but several periods $T = 2\pi N/V$ are present. This regime is replaced at higher voltages (for example, at $V = 0.2$ —point 5 in Fig. 3) by a regime with a single period and the situation becomes similar to the case at low voltages.

We expect that the S shape of the I - V characteristic in the $V = \text{const}$ regime will be conserved even for temperatures far from T_c , because the only condition for its occurrence is the existence of two different critical currents $j_{c1} < j_{c2}$. While $V < V_1$ the current in the wire will be an oscillating function with maximal amplitude j_{c2} and minimal amplitude j_{c1} . It means that the time averaged current should be larger than j_{c1} . At $V > V_1$ we expect that j_{\max} decreases due to the same mechanisms as was discussed above and hence the average current should decrease. But the maximal current cannot decrease below j_{c1} and for $V > V_2$ both j_{\min} and j_{\max} have to increase with increasing voltage.

In our theoretical consideration we neglected the effect of thermo- and/or quantum-activated phase-slip centers. We believe that this is the reason why in the experiment no hysteresis was observed in the $I = \text{const}$ regime. The

larger these fluctuations the larger they will decrease the measured j_{c2}^{exp} and increase the measured j_{c1}^{exp} (see also the discussion of this question in Ref. [15]) and, in principle, they may suppress the hysteresis. But in the $V = \text{const}$ regime the situation is quite different. If the time needed for the appearance of a fluctuating phase-slip center is much larger than the period of oscillation of the current, then we can neglect their effect.

In conclusion, we presented the first experiment on the I - V characteristic in the voltage constant regime for narrow superconducting Pb and Sn wires. An unexpected S -shape response is found which is absent in current driven measurements. A theoretical analysis based on the generalized TDGL equations gives a qualitative explanation of this phenomenon which is driven by a highly nonlinear time response of the superconductor to electric fields leading to the periodic creation of PS centers. Furthermore, we predict an oscillatory I - V characteristic at low voltages which was not found in the experiment presumably due to the large length of the wire and hence the relatively large value of N_{\min} .

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