# Condition for the occurrence of phase slip centers in superconducting nanowires under applied current or voltage

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(Received 17 September 2003; revised manuscript received 30 October 2003; published 17 March 2004)

Experimental results on the phase slip process in superconducting lead nanowires are presented under two different experimental conditions: constant applied current or constant voltage. Based on these experiments we established a simple model which gives us the condition of the appearance of phase slip centers in a quasi-one-dimensional wire. The competition between two relaxations times (relaxation time of the absolute value of the order parameter $\tau_{|\psi|}$ and relaxation time of the phase of the order parameter in the phase slip center $\tau_\phi$) governs the phase slip process. Phase slips, as periodic oscillations in time of the order parameter, are only possible if the gradient of the phase grows faster than the value of the order parameter in the phase slip center, or equivalently if $\tau_\phi < \tau_{|\psi|}$.

DOI: 10.1103/PhysRevB.69.094512 PACS number(s): 74.25.Op, 74.20.De, 73.23.−b

## I. INTRODUCTION

Since the discovery of superconductivity it was expected that if the superconductor was subjected to a constant electric field superconductivity will inevitably be destroyed. The reason is that the superconducting electrons will be accelerated by the electric field and will reach a velocity above its critical velocity. However, if the sample is short enough or if the electric field exists only in a small part of the sample such that the path over which the Cooper pairs are accelerated are sufficiently short an electric field can exist in the sample in the presence of superconductivity. Another example is the presence of an electric field in the superconducting sample which is attached to a normal metal. In this geometry the injected current from the normal metal will be converted into a superconducting current at a distance of about the charge imbalance distance ($\Lambda_Q$), and on that scale an electric field will exist in the sample.[1] In this case the electric field is compensated for by the gradient of the chemical potential $\mu_s$ of superconducting electrons and it does not lead to an acceleration of the superconducting condensate (see, for example, the book by Schmidt[2]).

But there is another mechanism which allows superconductivity to survive in the presence of an electric field deep in the superconducting sample of arbitrary length. This is the *phase slip* mechanism. Initially this phenomenon was used in order to estimate the relaxation time of superconducting current in a superconducting wire.[3] If the order parameter vanishes in one point of the wire the phase of the superconducting order parameter exhibits a jump of $2\pi$ at that point,[3] and as a result the momentum $p \simeq \nabla\phi$ decreases by $2\pi/L$. So even if the electric field accelerates the superconducting electrons it does not lead to a destruction of superconductivity because the momentum is able to relax through the phase slip mechanism.

This simple idea was already understood a long time ago. Therefore, it is very surprising that there are practically no experimental or theoretical works studying in detail what will happen if a voltage (i.e., electric field) is applied to the superconductor. In previous works mainly the situation with applied current (i.e., the $I=$const regime) was studied. In the latter case the phase slip process was studied theoretically in detail (see, for example, the review of Ivlev and Kopnin[4] and the books by Tinkham[1] and Tidecks[5]). On the basis of a numerical solution of the extended time-dependent Ginzburg-Landau equations it was found that the phase slip (PS) phenomenon exists in some region of currents. The lowest critical current at which a PS solution first appears in the system may be smaller than the depairing Ginzurg-Landau current density, and as a result it leads to hysteresis in the *I-V* characteristics in the $I=$const regime.[4,5]

The $I=$const regime was also studied experimentally in a number of works[4,5] starting from the paper of Meyer and co-workers.[6,7] The most characteristic effect of the phase slip mechanism is the appearance of a stair like structure in the current-voltage characteristics. In Ref. 8 a simple phenomenological model was built in order to quantitatively describe this feature. It was proposed that every step in the *I-V* characteristic is connected with the appearance of a new phase sip center (PSC) that increases the resistivity of the sample by a finite value. In this model, it was conjectured that in a region of the sample with size of about the coherence length $\xi$ fast oscillations of the order parameter occurs in time which produces normal quasiparticles. Because of the finite time needed to convert normal electrons into superconducting electrons[1,2] there is a region of size of about $\Lambda_Q$ near the phase slip center where the electric field and the normal current are different from zero. This means that the chemical potential of superconducting $\mu_s$ and normal electrons $\mu_n$ are different near the PSC and their difference is proportional to the charge imbalance $Q$ in a given point of the superconductor.[1,8] Usually $\Lambda_Q \gg \xi$, and therefore it is possible to neglect the region of size $\xi$ over which $|\psi|$ oscillates, which allows us to consider that phenomena as a time-independent process. In such a model every phase slip center contributes to a finite voltage, which is equal to $V_{PSC} = 2\Lambda_Q \rho_n (I-\beta I_c)/S$ with $\rho_n$ the normal resistivity, $S$ the cross section of the current carrying region, $I_c$ the critical current, and $\beta < 1$ a phenomenological parameter. In the experiment of Dolan and Jackel[9] the distribution of $\mu_s$ and $\mu_n$ near a PSC were measured which fully supported the idea of Ref. 8 that a difference exist between $\mu_s$ and $\mu_n$ near the phase slip center.

Despite numerous theoretical and experimental investigations the physical conditions under which phase slip phenomena can exist is still not clear. In a review[4] on this subject it was claimed that phase slip phenomena are connected with the presence of a limiting cycle in the system which leads to such types of oscillations. But up to now no explanation of how and why such a limiting cycle leads to phase slip has been presented.

Based on our previous work[10] on the time-dependent Ginzburg-Landau equations (the investigated systems were superconducting rings in the presence of an external magnetic field) we know that systems which are governed by such equations exhibit two relaxation times. One is the relaxation time of the phase of the order parameter $\tau_\phi$, and the other is the relaxation time of the absolute value of the order parameter $\tau_{|\psi|}$. In Ref. 10 it was established that phase slip processes can occur in such systems when, roughly speaking, $\tau_\phi < \tau_{|\psi|}$. In the present paper we discuss this question in the context of superconducting wires in the presence of an applied current or voltage.

To our knowledge there exists only a single theoretical work in which a superconducting wire in the presence of an applied voltage was studied.[11] The authors used the simple time-dependent Ginzburg-Landau equations and found that the behavior of the system is very complicated and strongly depends on the length of the wire and the applied voltage. Neither a detailed analysis nor any physical interpretation of their results was presented. In a recent letter[12] we presented our preliminary theoretical and experimental results on the dynamics of the superconducting condensate in wires under an applied voltage. It turned out that in this case the $I$-$V$ characteristics exhibit a S shape which was explained by the appearance of phase slip centers in the wire and their rearrangement in time. In the present paper we will present more details and extend our previous work to the situation in which defects are present, and we investigate the effects of boundary conditions and of an applied magnetic field.

The paper is organized as following. In Sec. II we present our theoretical results and give the conditions for the existence of phase slip centers when current (Sec. II A) or voltage (Sec. II B) is applied to the superconducting wire. In Sec. III we show our experimental results, and in Sec. IV we compare theory and experiment.

## II. THEORY

We study the current-voltage characteristics of quasi-one-dimensional superconductors using the generalized time-dependent Ginzburg-Landau (TDGL) equation. The latter was first written down in the work of Ref. 13:

$$\frac{u}{\sqrt{1+\gamma^2|\psi|^2}}\left(\frac{\partial}{\partial t}+i\varphi+\frac{\gamma^2}{2}\frac{\partial|\psi|^2}{\partial t}\right)\psi = (\nabla-\mathrm{i}\mathbf{A})^2\psi+(1-|\psi|^2)\psi. \quad (1)$$

In comparison with the simple time-dependent Ginzburg-Landau equation where $\gamma=0$, this allows us to describe a wider current region (with a proper choice of the parameters $u$ and $\gamma$) where a superconducting resistive state exists and gives us a wider temperature region in which Eq. (1) is applicable.[13–15] The inelastic collision time $\tau_E$ for electron-phonon scattering is taking into account in the above equation through the temperature dependent parameter $\gamma = 2\tau_E\Delta_0(T)/\hbar$ ($\Delta_0 = 4k_B T_c u^{1/2}(1-T/T_c)^{1/2}/\pi$ is the equilibrium value of the order parameter).

Equation (1) should be supplemented with the equation for the electrostatic potential

$$\Delta\varphi = \mathrm{div}\{\mathrm{Im}[\psi^*(\nabla-\mathrm{i}\mathbf{A})\psi]\}, \quad (2)$$

which is nothing else than the condition for the conservation of the total current in the wire, i.e., $\mathrm{div}\mathbf{j}=0$. In Eqs. (1) and (2) all the physical quantities (order parameter $\psi=|\psi|e^{i\phi}$, vector potential $\mathbf{A}$ and electrostatical potential $\varphi$) are measured in dimensionless units: the vector potential $\mathbf{A}$ and momentum of superconducting condensate $\mathbf{p}=\nabla\phi-\mathbf{A}$ is scaled by the unit $\Phi_0/(2\pi\xi)$ (where $\Phi_0$ is the quantum of magnetic flux), the order parameter is in units of $\Delta_0$ and the coordinates are in units of the coherence length $\xi(T)$. In these units the magnetic field is scaled by $H_{c2}$ and the current density by $j_0 = c\Phi_0/8\pi^2\Lambda^2\xi$. Time is scaled in units of the Ginzburg-Landau relaxation time $\tau_{GL} = 4\pi\sigma_n\lambda^2/c^2 = 2T\hbar/\pi\Delta_0^2$, the electrostatic potential ($\varphi$), is in units of $\varphi_0 = c\Phi_0/8\pi^2\xi\lambda\sigma_n = \hbar/2e\tau_{GL}$ ($\sigma_n$ is the normal-state conductivity). In our calculations we mainly made use of the bridge geometry boundary conditions: $|\psi(-L/2)| = |\psi(L/2)| = 1$, $\varphi(-L/2)=0$, $\varphi(L/2)=V$, and $\psi(L/2,t+dt)=\psi(L/2,t)e^{-i\varphi(L/2)dt}$. We chose these boundary conditions because at low temperatures the normal current is converted to a superconducting one due to the Andreev reflection on a distance of about $\xi_0 \simeq 0.18\hbar v_F/k_B T_c$ near the $S-N$ boundary.[1,17] This means that there is practically no injection of quasiparticles from the normal material to the superconductor and hence we can neglect the effect of charge imbalance near the S-N boundary. The bridge geometry boundary conditions models this situation. The parameter $u$ is about 5.79 according to Ref. 13. We also put $A=0$ in Eqs. (1) and (2) because we considered the one-dimensional model, in which the effect of the self-induced magnetic field is negligible and we assume that no external magnetic field is applied.

### A. Constant current regime

Let us first consider the more simple case when a constant external current is applied to the sample. In such a case it was theoretically found[13,14] that the system exhibits a hysteretic behavior. If one starts from the superconducting state and increases the current the superconducting state switches to the resistive superconducting or normal state at the upper critical current density $j_{c2}$ which, in a defectless sample, is equal to the Ginzburg-Landau depairing current density $j_{GL} = \sqrt{4/27}j_0$. Starting from the resistive state and decreasing the current it is possible to keep the sample in the resistive state even for currents up to $j_{c1} < j_{c2}$ (which we call the low critical current). For $j_{c1} < j < j_{c2}$ such a state is realized as a periodic oscillation of the order parameter in time at one point of the superconductor.[13,14] When the order parameter reaches zero in this point a phase slip of $2\pi$ occurs. This is the reason why such a state is now called a phase slip state and this point a phase slip center (PSC). Using results obtained [on the basis of Eq. (1) with $\gamma=0$] in our earlier work[10] we claim that the value of $j_{c1}$ depends on the ratio between the two characteristic times in the sample: the phase relaxation time of the order parameter $\tau_\phi$ and the relaxation time of the absolute value of the order parameter $\tau_{|\psi|}$ in the region (with size of about $\xi$) where the oscillations of the order parameter occurs. The reasons for this are as follows.

Having written Eq. (1) for the dynamics of the phase and the absolute value of the order parameter in a quasi-one-dimensional wire of length $L$ $(-L/2 < s < L/2)$

$$u\sqrt{1+\gamma^2|\psi|^2}\,\frac{\partial|\psi|}{\partial t} = \frac{\partial^2|\psi|}{\partial s^2} + |\psi|[1-|\psi|^2-(\nabla\phi)^2], \tag{3a}$$

$$\frac{\partial\phi}{\partial t} = \varphi - \frac{\sqrt{1+\gamma^2|\psi|^2}}{u|\psi|^2}\frac{\partial j_n}{\partial s}, \tag{3b}$$

it is easy to estimate both relaxation times. Indeed, from Eq. (3a) it directly follows that

$$\tau_{|\psi|} \sim u\sqrt{1+\gamma^2|\psi|^2} \simeq u\gamma \quad (\text{for} \quad \gamma \gg 1). \tag{4}$$

To determine $\tau_\phi$ we need to know how fast the phase (or more exactly the phase gradient $\nabla\phi$) changes over the region (with size of about $\xi$) at which the order parameter oscillates. Because the electrostatic potential $\varphi$ changes over a distance $\Lambda_Q \gg \xi$ for $\gamma \gg 1$ (where $\Lambda_Q^2 = \sqrt{1+\gamma^2|\psi|^2}/u|\psi|^2 \simeq \gamma/u$ is the square of the decay length of the normal current density and the charge imbalance $Q$) and that the order parameter is about unity already at $s = \pm\xi$, we can estimate the time derivative $\partial(\phi_{+\xi} - \phi_{-\xi})/\partial t \sim \Lambda_Q j_n(0)$. On first sight we could also use the superconducting component $j_s = |\psi|^2\nabla\phi$ of the full current in order to estimate the left hand side (LHS) of Eq. (3b) due to the relation $\partial j_n/\partial s = -\partial j_s/\partial s$. But near the phase slip center we cannot use approximation $\partial j_s/\partial s \sim j_s(0)\lambda_Q$ because both $|\psi|$ and $\phi$ change appreciably on a length scale of $\xi$. Therefore it is essential that the LHS of Eq. (3b) is proportional to the normal current density in the phase slip center.

Consequently we find that

$$\tau_\phi \simeq \frac{1}{\Lambda_Q j_n(0)}. \tag{5}$$

In terms of the language used by Schmid and Schön,[18] $\tau_{|\psi|}$ is the "longitudinal" time and $\tau_\phi$ decreases with increasing "transverse" time because $\tau_\phi \sim 1/\Lambda_Q \sim 1/\sqrt{\tau_Q}$.

In Ref. 10 the transitions between states with different vorticity were studied for a superconducting ring in a perpendicular magnetic field. For that purpose Eq. (1) with $\gamma=0$ was used. It turned out that transitions occur through the appearance of phase slip centers in some point along the ring. The main result of that paper can be summarized as follows: In the point where the phase slip event occurs the absolute value of the order parameter initially decreases and after the phase slip event $|\psi|$ increases. This means that the right hand side (RHS) of Eq. (3a) is negative before the phase slip event and positive after the phase slip event in the phase slip region. In Ref. 10 it was shown that the distribution of $|\psi|$ near the phase slip center is almost the same just before and after the phase slip event and that the sign of the RHS of Eq. (3a) is governed by the competition between the positive term $\partial^2|\psi|/\partial s^2 + |\psi|(1-|\psi|^2)$ and the negative term $-(\nabla\phi)^2|\psi|$. It turned out that there is a critical value $(\nabla\phi)_c$ which depends on the value of the order parameter in the phase slip center $|\psi|_{PSC}$ such that if $\nabla\phi > (\nabla\phi)_c$ the RHS of Eq. (3a) is negative which provides the necessary condition for the appearance of the next phase slip event (for further details, see Ref. 10). It is essential that the time over which $|\psi|$ and $\nabla\phi$ changes are different [see Eqs. (4) and (5)]. Thus if after the phase slip event $|\psi|$ increases faster than $\nabla\phi$ the next phase slip event will be impossible because at any time $\nabla\phi$ will be less than $(\nabla\phi)_c$. This leads us to conclude that phase slip process which is periodic in time can exist, but to the condition that the inequality $\tau_\phi \lesssim \tau_{|\psi|}$ is fulfilled. Because the above consideration may give us only an order-of-magnitude estimate, we can with the same accuracy replace $j_n(0)$ by an applied current $j$ [see Fig. 1(b) below] in Eq. (5). This gives us an estimation for the lower critical current:

$$j_{c1} \simeq \frac{1}{\tau_{|\psi|}\Lambda_Q}. \tag{6}$$

By varying the parameter $\gamma$ we can change both $\tau_{|\psi|}$ and $\Lambda_Q$ and hence we can vary $j_{c1}$. Already in Ref. 13–15 it was found that by increasing the parameter $\gamma$ (or $u$ as was done in Ref. 4) leads to a decreasing lower critical current $j_{c1}$. This is direct consequence of Eq. (6) because $\tau_{|\psi|}\Lambda_Q \sim \gamma^{3/2}u^{1/2}$. In Fig. 1 we show the voltage in the sample, the normal current density and the absolute value of the order parameter in the phase slip center averaged over the time as a function of the external current for two different values of $\gamma$. In our simulations we started from the superconducting state with $j < j_{GL}$. At $j > j_{GL}$ the system jumps instantaneous to the resistive state with finite voltage.[19] Than we decrease the external current and at $j < j_{c1}$ the system transits to the purely superconducting state with $V=0$. We should emphasize that at $j \to j_{c1}$ the voltage jumps by a *finite* value (this result is qualitatively different from the results of Refs. 13–15, where the authors found that $V \to 0$ at $j \to j_{c1}$). It means that there is a *finite* maximal oscillation period for the order parameter in the phase slip center. We believe that the finite voltage jump $\Delta V$ or finite period of oscillations is directly connected

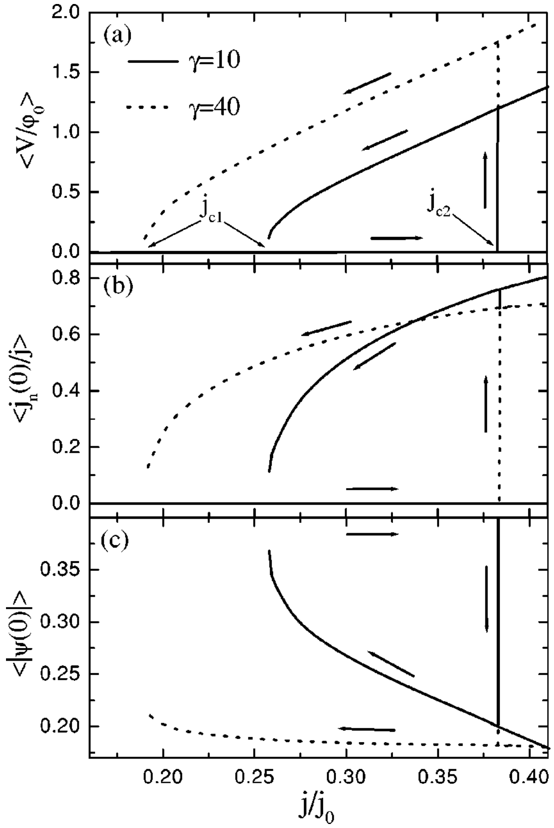


FIG. 1. The dependence of the time averaged voltage (a) on the external current for a wire containing only one phase slip center. In (b) and (c) the dependencies of the normal current density (b) and the order parameter (c) in the phase slip center are shown. Length of the wire is $40\xi$. Solid curves correspond to $\gamma=10(\Lambda_Q\simeq2.3\xi)$, and the dotted curves are for $\gamma=40$ $(\Lambda_Q\simeq4.1\xi)$.

with the threshold condition $\tau_\phi\simeq\tau_{|\psi|}$ for the activation of regular phase slip processes in the constant current regime and it means that $\Delta V\sim1/\tau_{|\psi|}$.

Before going further we should stress here that the condition $\tau_\phi\lesssim\tau_{|\psi|}$ for the existence of a phase slip process is a rather rough estimate. Indeed, Eqs. (3a) and (3b) are a coupled system of equations, and besides $\tau_{|\psi|}$, it explicitly [see Eq. (4)] depends on the value of $|\psi|$ and $\tau_\phi$ on the normal current density in PS center [see Eq. (5)]. However, the above condition allows us to explain the general qualitative properties of the phase slip processes (including the existence of $\Delta V$ and $j_{c1}$ and their dependence on $\gamma$ and $u$) and predict new features which will be discussed below.

Far enough from $j_{c1}$ the dependence of $V(j)$ on the current is close to linear. When the current increases, $\tau_\phi$ decreases and hence the time $\tau_{PSC}$ between two phase slips also decreases and the order parameter has less time for recovering at the phase slip center. That is the reason why the time averaged voltage increases (as $\langle V\rangle=2\pi/\tau_{PSC}$), the averaged order parameter $\langle|\psi(0)|\rangle$ decreases and the fraction of the normal current $\langle j_n(0)\rangle/j$ increases with increasing external current. It is interesting to note that from the phenomenological Skocpol-Beasley-Tinkham (SBT) (Ref. 8) model it follows that $\langle j_n(0)\rangle/j=1-\beta j_{c1}/j$ which qualitatively resembles the dependence shown in Fig. 1(b).

Let us now discuss the effect of defects on the *I-V* characteristic. This question was considered previously in Ref. 20 for two different models of defects: a local variation of the critical temperature and a local variation of the mean free path. We will repeat these calculations partially and interpret it in terms of a competition between $\tau_\phi$ and $\tau_{|\psi|}$. In addition to the first type of defect we will also study the effect of the variation of the cross-section of the wire.

The first type of defect is the inclusion of a region in the superconductor which suppresses $T_c$ and the order parameter becomes lower than the equilibrium value $\Delta_0$ even in the absence of any external current. This is modeled[20] by introducing the term $\rho(s)\psi$ on the RHS of Eq. (1). In the present calculation we choose $\rho(s)=-\rho_0\theta(0.5-|s|)$. The larger $\rho_0$ is the more the order parameter is suppressed in the center of the wire. First, such a defect leads to a decrease of the upper critical current $j_{c2}$. Secondly, it decreases the lower critical current $j_{c1}$. Indeed, when we introduce a defect, the RHS of Eq. (3a) decreases at the spatial position where the order parameter oscillations and hence $|\psi|$ needs more time to change in that spatial position. Thus this type of defect leads to an increase of the relaxation time of the order parameter in the region of the defect (as an indirect proof of this we found a decreasing $\Delta V$ with an increasing "strength" of the defect). If the size of the defect is smaller than $\Lambda_Q$ we can neglect its effect on the relaxation time of the phase of the order parameter. Finally, we can conclude that the "stronger" the defect the smaller the value of the lower critical current $j_{c1}$.

In Fig. 2(a) we present our numerical results for two different values of $\rho_0$. With an increasing strength of the defect the upper and lower critical currents decrease and start to merge. As a consequence the hysteresis in the *I-V* characteristics will disappear when the defect is sufficiently strong.

The second type of defect is one for which we have a local decrease of the cross-section of the wire. In this case we expect that $\tau_{|\psi|}$ will not be influenced. The situation with $\tau_\phi$ is more complicated because a large part of the superconductor with a size of about $\Lambda_Q$ participates in the formation of this time. Two limiting cases can be distinguished. If the defect is much smaller than $\Lambda_Q$ we can neglect its effect on $\tau_\phi$, and hence we will have the same lower critical current as for the case of an ideal wire. In the opposite case of a defect with a size much larger than $\Lambda_Q$ we only have to take into account the increased current density in the part of the wire where we have a smaller cross-section and apply Eq. (5). In this case the current $j_{c1}$ is decreased by a factor $D_{av}/D_l$, where $D_{av}$ is the average cross-section and $D_l<D_{av}$ is the local reduced cross section.

Another important question is the value of the upper critical current $j_{c2}$. If the size of the defect is larger than $\xi$ then the proximity effect from adjacent parts near the defect will be small and $j_{c2}$ will decrease by a factor $D_{av}/D_l$ (in the opposite case the current $j_{c2}$ is almost defect independent). Therefore, for a defect with length $\xi<l\ll\Lambda_Q$ the *I-V* characteristic may be reversible with a proper choice of the parameters (as in the case of a local suppression of $T_c$). In Fig. 2(b) we show the results of our numerical calculations and

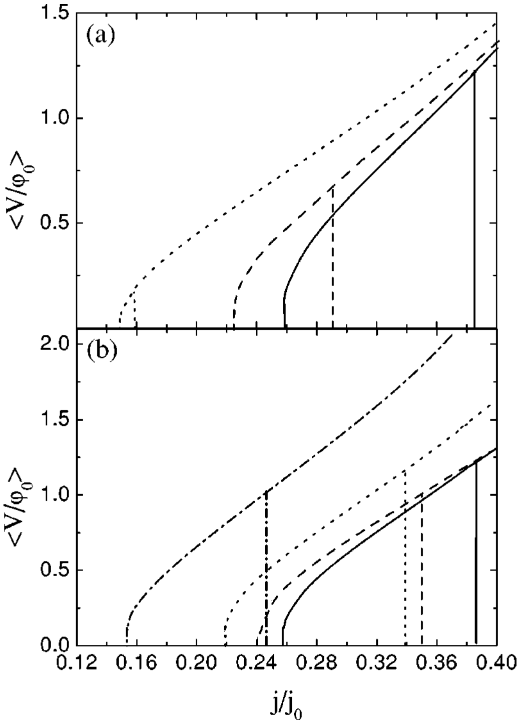

FIG. 2. The dependence of the time averaged voltage on the external current for a wire containing a single defect. (a) corresponds to a wire with a local suppression of the $T_c$ (dotted curve for $\rho_0=-2$, dashed curve for $\rho=0$ and solid curve for a wire without defect). (b) corresponds to a local variation of the cross section of the wire which we modeled as $D(s)=1-\beta e^{-s^2/l_d^2}$. The solid curve in this figure is for a wire without a defect, the dashed curve for a wire with defect parameters $\beta=0.5$ and $l_d=0.5$, the dotted curve with $\beta=0.2$ and $l_d=2$, and the dash-dotted curve with $\beta=0.5$ and $l_d=2$. The length of the wire is $L=40\xi$ and the parameter $\gamma=10$ ($\Lambda_Q\simeq 2.3\xi$).

we obtain a qualitative agreement with the above physical arguments. When the length of the defect is smaller than $\Lambda_Q$ then the lowest critical current density practically does not change. In the opposite case $j_{c1}$ decreases by a factor of $D_{av}/D_l$. The upper critical current density changes considerably only if the length of the defect exceeds $\xi$. Unfortunately, we are not able to consider the case for which the length of the defect is simultaneously much smaller than $\Lambda_Q$ and much larger than $\xi$ because of computational restrictions. For example, when we increase $\gamma$ by a factor of 2 the time of calculations also increases by a factor of 2 but the ratio $\Lambda_Q/\xi$ increases roughly only by a factor of $\sqrt{2}$ for $\gamma \gg 1$. The reason for this is that when we increase $\gamma$ we need to take a smaller time step [see Eq. (1)] which increases the computation time.

When the wire has a length less than $\Lambda_Q$ then this will affect the distribution of the normal current density in the wire and hence the time $\tau_\phi$ and the lower critical current $j_{c1}$. Indeed from our bridge boundary conditions it follows that $\partial j_n/\partial s(\pm L/2)=0$ [see Eq. (3b)]. Then taking into account that $\partial\langle\phi\rangle/\partial t=V\theta(s)/2$ we can easily obtain, in the limit $\Lambda_Q \gg \xi$, that

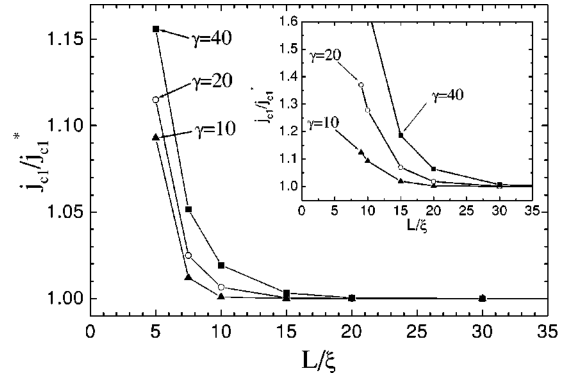

FIG. 3. Dependence of the low critical current $j_{c1}$ on the length of the superconducting wire for different values of $\gamma$. The current $j_{c1}$ is normalized to its value ($j_{c1}^*$) at lengths $L\gg\Lambda_Q$. With increasing $\gamma$ the decay length of the normal current increases and hence $j_{c1}$ starts to depend on $L$ for longer wires. In the inset, the dependence of $j_{c1}$ on the length of the superconducting wire for S-N boundary conditions is shown.

$$\langle j_n(0)\rangle = \frac{V(j)}{2\Lambda_Q}\frac{1}{\tanh(L/2\Lambda_Q)}, \tag{7}$$

where the current $\langle j_n(0)\rangle$ is the average normal current in the phase slip center.

The voltage jump $\Delta V \simeq 2\pi/\tau_{|\psi|}$ at $j_{c1}$ should not change with varying $L$ (at least for $L\gg\xi$ when the proximity effect does not have an effect on $\tau_{|\psi|}$). From Eq. (7) it then follows that the normal current density increases with increasing length. But $\langle j_n(0)\rangle$ should always be smaller than the full current $j$. This implies that the critical current density $j_{c1}$ will increase with the decreasing length of the wire as $j_{c1} \sim 1/\tanh(L/2\Lambda_Q)$ in order to keep the ratio $\langle j_n(0)\rangle/j$ constant. Figure 3 illustrates the dependence of $j_{c1}$ on the length of the wire which we obtained on the basis of a numerical solution of Eqs. (1) and (2). Unfortunately, in our calculations we are not able to use very large values of $\gamma$ and the maximal value of $\Lambda_Q$ was $4.1\xi$ for $\gamma=40$. But nevertheless, we found that $j_{c1}$ increases with decreasing wire length. We should add here that we also found that $\Delta V$ decreases a little bit. We connect this with a small change in $\tau_{|\psi|}$ due to the small length of the wire and hence the increased effect of the boundaries.

A more pronounced effect of the finite length of the wire on the value of $j_{c1}$ is found for the case that we use the N-S boundary conditions: $\psi(\pm L/2)=0$ and $\partial\varphi/\partial s(\pm L/2) = -j$. These boundary conditions are approximately valid for samples at temperatures close to $T_c$. It is easy to show that in this case

$$\langle j_n(0)\rangle = \frac{V(j)}{2\Lambda_Q}\frac{\tanh(L/2\Lambda_Q)}{1-1/\alpha\cdot\cosh(L/2\Lambda_Q)}, \tag{8}$$

where $\alpha=\langle j_n(0)\rangle/j<1$ [see Fig. 1(b)]. In this case $j_{c1}$ also increases with decreasing wire length (see the inset in Fig. 3). In addition, there is a finite length $L_0$ for which $\langle j_n(0)\rangle \to\infty$. This implies that the phase slip process is not possible in wires with length $L<L_0$. In such wires the system goes from the superconducting state directly to the normal state.

Not only the finite length of the sample is able to change the value of $j_{c1}$. If we apply a magnetic field parallel to the length of the wire it will suppress the order parameter in the sample. Our analysis shows that if the diameter of the wire $d$ is less than $2\xi$ and if we can neglect screening effects ($\lambda > \xi$) then the distribution of the order parameter will be uniform along the cross-section of the wire. The order parameter depends on $H$ as

$$|\psi|^2 = 1-(H/H_c)^2$$

with $H_c \simeq 2.9\Phi_0/\pi\xi d$. This behavior is very similar to the behavior of a thin plate in a parallel magnetic field[21,22] or a thin and narrow ring in a perpendicular magnetic field.[23] In all cases the transition to the normal state is of second order and the vorticity in the wire will be equal to zero due to the small cross section of the sample.

Because the order parameter practically does not depend on the radial coordinate of the wire we can use the one-dimensional model in order to study the response of the system on the applied current. In order to take into account the suppression of $|\psi|$ by the magnetic field ($H$) we add to the RHS of Eq. (1) the term $-(H/H_c)^2\psi$. In some respects this is similar to the way we introduced the first type of defect in our wire. We can expect that $\tau_{|\psi|}$ will increase with increasing $H$ (because the strength of the defect increases). But because the magnetic field suppresses the order parameter everywhere in the sample it also leads to an increase of $\Lambda_Q$, because in the TDGL model $\Lambda_Q \sim 1/\sqrt{|\psi|}$. It is clear that both these processes should decrease $j_{c1}$. The strongest mechanism is connected with the change in $\tau_{|\psi|}$. Indeed it is easy to estimate that $\tau_{|\psi|} \simeq 1/[1-(H/H_c)^2]$ and $\tau_\phi \simeq [1-(H/H_c)^2]^{1/4}$. Even if we take into account that the parameter $\gamma$ may decrease with increasing $H$ (because in a nonzero magnetic field there is another pair-breaking mechanism and instead of $\tau_E$ we should use[16,18] $\tau_E/\sqrt{1+2\tau_E\tau_s}$ with $\tau_s = \Delta(T=0,H=0)/\hbar[H/H_c(T=0)]^2$) it does not lead to an increase of $j_{c1}$ with an increase of $H$ because $\tau_{|\psi|}$ changes faster than $\tau_\phi$ even in this case.

In the above model it is easy to show that $j_{c2}(H) = \sqrt{4/27}[1-(H/H_c)^2]^{1.5}$ for the case of an uniform wire. In Fig. 4 we present the results of our numerical calculations. We found that both $j_{c1}$ and $j_{c2}$ decrease with an increasing magnetic field, and at some $H^*$ they practically merge. Unfortunately, it is quite difficult to find an analytical expression for the dependence of $j_{c1}(H)$ like we had for $j_{c2}$. The reason is the complicated behavior of the dynamics of $\psi$ in the phase slip center.

The variation $\Lambda_Q$ with increasing $H$ was obtained experimentally in Ref. 16. To compare with the theory the authors of Ref. 16 used the expressions found in the work of Schmid and Schön[18] which are valid in the limit $T \to T_c$. It is interesting that in Ref. 16 it was found that expressions of Schmid and Schön are quantitatively valid even far from $T_c$. From this observation we may hope that the present theoretical results are also valid over a wider temperature range than only near $T_c$.

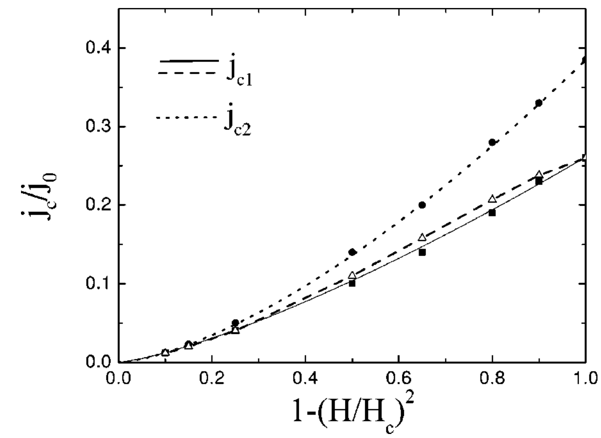


FIG. 4. Dependence of the lower critical current $j_{c1}$ (squares and solid curve) and upper critical current $j_{c2}$ (dots and dotted curve) on the applied magnetic field. Dotted curve is equation $\sqrt{4/27}[1-(H/H_c)^2]^{1.5}$. The solid curve is the fitted expression $0.26\cdot[1-(H/H_c)^2]^{1.36}$. We also plotted the dependence $j_{c1}(H)$ (open triangles and dashed curve) where the effect of $H$ on $\gamma$ was taken into consideration. We used the simple expression $\gamma_{eff} = \gamma_0/\sqrt{1+\gamma_0(H/H_c)^2}$ which even overestimates the effect of $H$ (i.e., we took $\gamma_0 = \gamma(H=0)$).

The main conclusion which follows from the change of $j_{c1}$ with decreasing wire length and/or applying magnetic field is that there exist a critical length $L^*$ (or critical field $H^*$) below (above) which the current $j_{c1}$ becomes equal to $j_{c2}$. It implies that for wires with lengths $L<L^*$ and/or fields $H>H^*$ there will be no jump in the voltage in the current-voltage characteristic and the *I-V* curve will be reversible. It will also result in the absence of a S-behavior in the $V=$const regime (see Sec. II B).

### B. Constant voltage regime

In our earlier work we found that the *I-V* characteristic in the $V=$const regime exhibits an S-like behavior[12] and furthermore for low voltages there is an oscillatory dependence of the current on the applied voltage (see Fig. 5). As was shown in Ref. 12 these properties are connected with the existence of two critical currents $j_{c1}$ and $j_{c2}$. Here, we will discuss the characteristic voltages $V_1$ and $V_2$ (see Fig. 5) and their dependence on the length of the sample.

If we apply a voltage $V$ to the wire of length $L$, then in the sample, an electric field $E=V/L$ exist which will accelerate the superconducting electrons.[24] When the current density approaches $j_{c2}$ phase slip centers will spontaneously appear in the sample. As a result the momentum of the superconducting electrons (and hence the current) will decrease by $2\pi/L$ after each phase slip event. When the current density decreases below $j_{c1}$ the phase slip process will no longer be active and the applied electric field will be able to accelerate the superconducting condensate. This process leads to periodic oscillations in time of the current in the sample.

We can divide the period of oscillations (at least at voltages $V<V_1$) in two parts. First, the longest part is the one during which the condensate is accelerated by the electric field till the moment reaches $j \simeq j_{c2}$. The second part we call

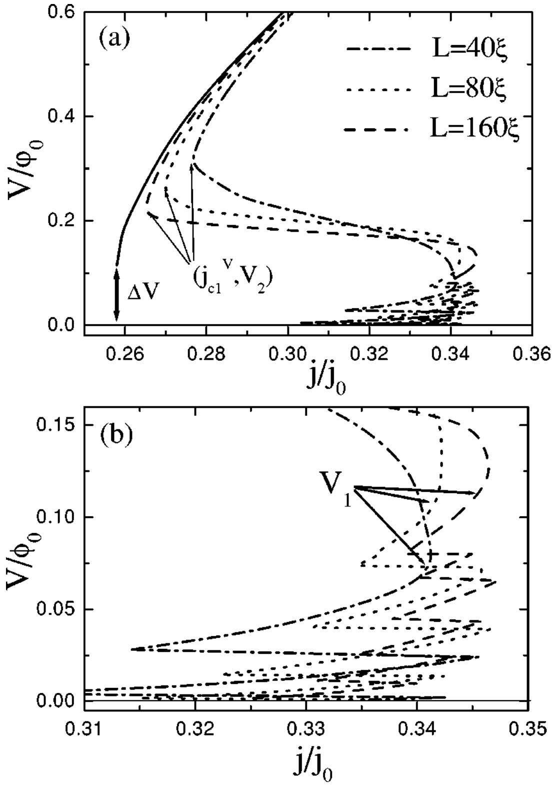


FIG. 5. Theoretical current-voltage characteristics of wires of different lengths for $\gamma=10$. The solid curve corresponds to the $I=\mathrm{const}$ regime and is practically universal for the considered lengths. The dot-dashed curve ($L=40\xi$), dotted curve ($L=80\xi$), and dashed curve ($L=160\xi$) correspond to the $V=\mathrm{const}$ regime. (b) is an enlargement of the low voltage region in (a).

the *transition period* $T_{tr}$. The transition period also consists of two parts: the time needed for the phase slip processes (which is proportional to the number of phase slip events and hence the length of the wire) and the time $T_0$ for the decay of the order parameter from $\sqrt{2/3}$ (when $j\simeq j_{c2}$) till the first phase slip event and for the recovering of $|\psi|$ back to $\sqrt{2/3}$ from zero after the last phase slip event (see Fig. 6). The minimal number of phase slip events which occur during the transition time is determined by the internal parameters of the superconductor ($j_{c1}$) and its length[12] which is given by

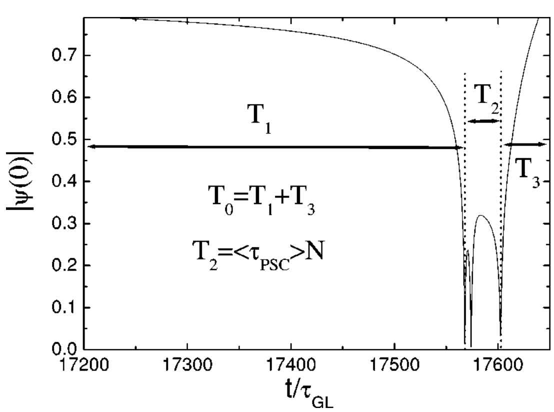


FIG. 6. The transition time consists of two parts: $T_{tr}=T_0+T_2$.

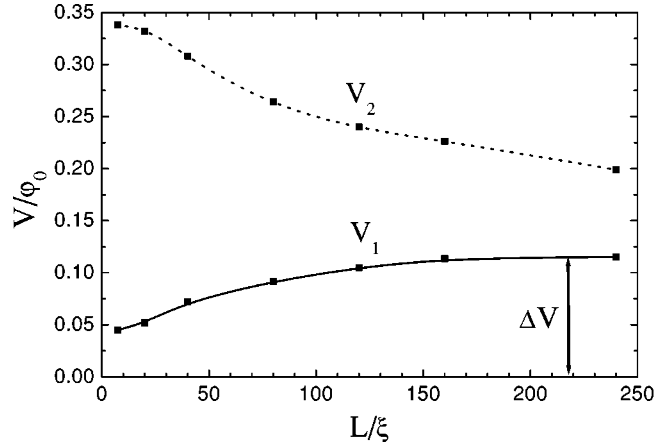


FIG. 7. Dependencies of the voltages $V_1$ and $V_2$ on the length of the wire. The results are obtained for $\gamma=10$.

$$N_{\mathrm{min}}=N_{\mathrm{int}}[(p_c-p_{c1})(L/2\pi+1)], \tag{9}$$

where $p_{c1}$ is the smallest real root of the equation $j_{c1}=p_{c1}(1-p_{c1}^2)$ and $N_{\mathrm{int}}(x)$ returns the nearest integer value.

At $V<V_1$ the time-averaged current $\langle j\rangle$ increases (with oscillations) and in the range $V_1<V<V_2$ it decreases with increasing voltage. The lowest minimal current in the latter region is $j_{c1}^V$ which depends on the length of the system (see Fig. 5 and Ref. 12). In Fig. 7 we present the dependence of the above voltages on the length of the wire. The explanation for their different behavior is the following. At a voltage $V_1$ the period of the oscillation $T=2\pi N/V_1$ ($N$ is the number of phase slip events during the transition time) of the current becomes of order $2T_{tr}$. The time $T_0$ does not depend on the length and the voltage at $V\sim V_1$. So, we can estimate $V_1$ as

$$V_1\simeq\frac{2\pi N}{2T_{tr}}=\frac{\pi N}{T_0+\langle\tau_{PSC}\rangle N}=\frac{\pi\alpha L}{T_0+\langle\tau_{PSC}\rangle\alpha L}, \tag{10}$$

where $\langle\tau_{PSC}\rangle$ is the average time between two phase slip events and $\alpha$ is the coefficient which depends on $p_{c1}$ [see Eq. (9)] and hence on the specific superconductor. When $T_0\ll\langle\tau_{PSC}\rangle\alpha L$ the voltage $V_1$ becomes practically independent of the length. Because the time $T_0$ depends on the relaxation time of the absolute value of the order parameter the length of the wire $L_{sat}$ at which $V_1$ saturates depends on the internal parameters of the superconductor.

The voltage $V_2$ decreases with increasing length of the sample because the lower critical current $j_{c1}^V$ decreases. It is interesting to note that the saturated value of the lower critical voltage coincides with the voltage jump $\Delta V$ at $j=j_{c1}$ in the constant current regime. Because the minimal value of $V_2$ is equal to $\Delta V$ for an infinitely long wire we may conclude that with increasing wire length the $I$-$V$ characteristic in the range of voltages $(V_1,V_2)$ approaches the horizontal line.

Defects, magnetic field, short length of the sample, etc. will decrease the hysteresis and consequently the currents $j_{c1}$ and $j_{c2}$ approaches each other. If the difference between them is small enough the S shape of the $I$-$V$ characteristic at the constant voltage regime changes to the usual monotonic

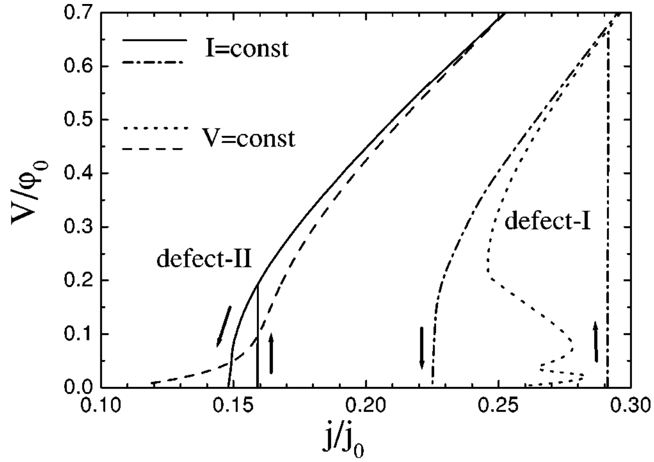


FIG. 8. Current voltage characteristics of a superconducting wire with a local suppression of the critical temperature in the center of the wire. The parameters for the defect and the wire are the same as in Fig. 2(a). For the *I*-*V* characteristic with small hysteresis in the $I=$const regime the *I*-*V* behavior in the $V=$const regime is a single-valued function of the current.

behavior (see Fig. 8). This happens because when the voltage approaches $V_1$, the maximal current during the period of the current oscillation $T$ will be about $j_{c1}$ (because $j_{max}$ cannot substantially exceed $j_{c2}$ but $j_{c2}$ is already close to $j_{c1}$ in this case).

We also would like to discuss how the change of the boundary conditions will change the shape of the *I*-*V* characteristic. If we apply the *N*-*S* boundary conditions the *I*-*V* curve in the voltage driven regime also exhibits an S behavior but without oscillations in the current at small voltages (see Fig. 9). In this case there will always be an inevitable voltage drop near the boundaries connected with the current in the wire through the relation $V_{NS}\sim j\Lambda_Q$ (because the electric field and the normal current decays on a scale of the charge imbalance length $\Lambda_Q$ near the boundaries[1]). Near the boundary the nonzero electric field is compensated by the term $\Lambda_Q^2\partial E/\partial s$ [see Eq. 3(b) and $E=j_n$ in our units] and the superconducting electrons are not accelerated by this field.

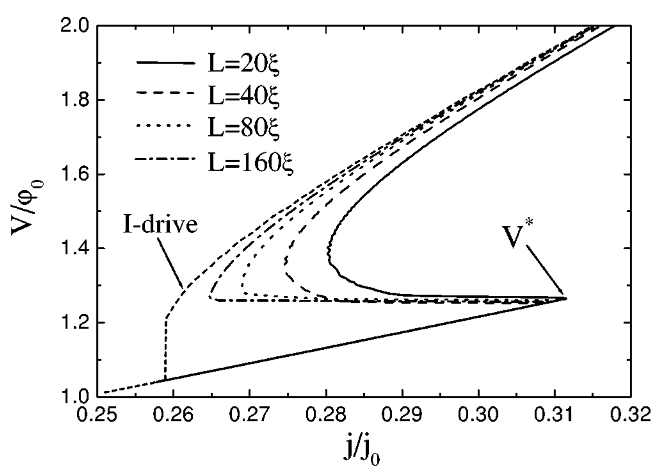


FIG. 9. Current-voltage characteristics of wires of different lengths in the case of the *N*-*S* boundary conditions.

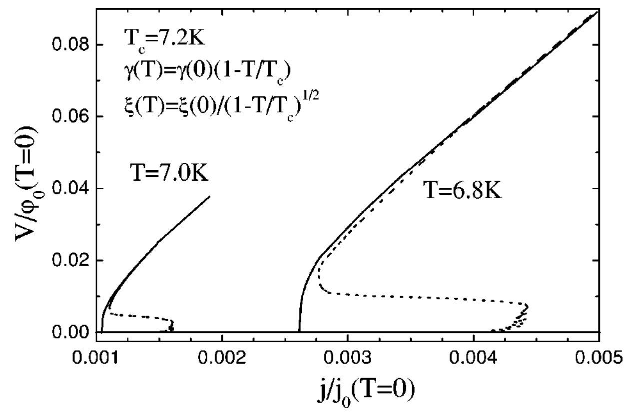


FIG. 10. Current voltage characteristics of a superconducting wire of length $600\xi(0)$ at different temperatures close to $T_c$. We used typical parameters for Pb: $\xi(0)\simeq 40$ nm and $\gamma(0)\simeq 100$. This result shows an increase of the difference $j_{c2}-j_{c1}$ in absolute units with decreasing temperature.

The situation is very similar to the case when we inject a current in the wire. When the current generated by this voltage reaches $j_{c1}$ the phase slip process becomes possible in the system. But as in the case of the current driven regime the superconducting state can be stable (metastable) until the current reaches $j_{c2}$ at $V\simeq j_{c2}\Lambda_Q$. From our estimates it follows (see Sec. III) that the fluctuations of the order parameter are very important in our samples. Therefore, in our calculations we introduce at some moment of time (at fixed voltage) a phase slip center in the center of the wire and checked if it will survive or not. As a result we obtained the *I*-*V* characteristics as presented in Fig. 9. When the voltage is less than some critical value ($V^*$) the phase slip process decays in time and the resulting current in the wire is time-independent. The whole voltage drop occurs near the boundaries. At $V>V^*$ (and $j>j_{c1}$) the dynamics of the condensate in the wire will be similar to the case considered above for $V>V_1$.

The reason for this is as follows. The current density in the wire at small voltages will always be less than $j_{c1}$ and $j_{c2}$ due to the relation $j\sim V_{SN}/\Lambda_Q=V/\Lambda_Q$. When the current density reaches $j_{c1}$ the phase slip process becomes possible in the sample. But such a phase slip process leads to a finite voltage. As a consequence the voltage drop at the boundaries will sharply decrease when a phase slip center is created. But it implies that the full current density will also decrease and consequently will become less than $j_{c1}$. We can conclude that the phase slip process may survive in such a type of superconductor only if the applied voltage will be roughly equal to $\Delta V(j)$ plus the voltage drop near the boundaries $V_{SN}$ necessary for the creation of a current larger than $j_{c1}^V$.

Concluding the theoretical part, we present in Fig. 10 the *I*-*V* characteristics in the $I=$const and $V=$const regimes at different temperatures close to $T_c$. With decreasing temperature the range of currents, where there is a S-behavior increases and the voltages $V_1$ and $V_2$ increases. It resembles the experimental results presented in Refs. 12 and 25 and in Sec. III.

TABLE I. Parameters for the different samples.

| | $L(\mu m)$ | $d$(nm) | $R_n(7.1K)(\Omega)$ | $R_{res}(4.3K)(\Omega)$ | $H_c(4.5K)(T)$ | $\xi_1(4.5K)$(nm) | $\xi_2(4.5K)$(nm) | $\rho_n(7.1K)(\mu\Omega\cdot cm)$ |
|---|---|---|---|---|---|---|---|---|
| *A* | 22 | 40 | 300.9 | 14.9 | 1.271 | 37 | 16 | 1.67 |
| *B* | 50 | 55 | 210.2 | 21.7 | 0.925 | 38 | 19 | 0.86 |
| *C* | 50 | 55 | 465.9 | 80.7 | 1.652 | 21 | 14 | 1.75 |
| *D* | 50 | 70 | 94.6 | 17.6 | 0.591 | 46 | 24 | 0.52 |

## III. EXPERIMENT

Measurements of the *I-V* characteristics were done on single Pb nanowires[12,25] with typical diameter about 50 nm and length 22–50 $\mu$m (see Table I), applying either a dc current of a dc voltage. Conceptually, applying a voltage bias to a superconductor may appear difficult because it requires a bias source with a lower internal impedance. However, our superconducting nanowires always possess a nonzero residual resistance (around 20 $\Omega$) due to contact resistance and/or a resistance associated to the metallic part of the superconductor-metal-insulator (SMI) transition induced by disorder.[26] This residual resistance was measured independently with a lock-in amplifier by applying a very small current (0.1 $\mu$A). The initial slope of the *I-V* curve, measured when applying a dc current, also reflects this nonzero residual resistance (see Refs. 12 and 25). This allowed us to apply a voltage to such nanowires using a voltage source, which has an internal impedance as low as $\sim 0.1\ \Omega$. Moreover, the resulting dc applied voltage was then measured separately with a Keithley voltmeter and the current flowing into the nanowires was measured using a 1 $\Omega$ resistance added in series. We also checked that the loadlines of our voltage source can be described by the equation $V(\mathrm{mV}) = -2.135\times 10^{-3}\ \mathrm{I}(\mu\mathrm{A}) + \mathrm{const}$. The slope of these loadlines are substantially flatter than the slope of the ‘‘S-shaped’’ curves that we obtained in these voltage driven *I-V* characteristic measurements (see Fig. 13 below).

At first we would like to discuss the rather wide resistive transitions and strong magnetoresistance effect (see Figs. 11 and 12) in our samples. We may claim, taking into account the small diameter of our nanowires, that the effect of the thermoactivated[3] and the quantum-activated[27,28] phase slip phenomena is very strong in our samples. It is easy to see that with increase of the diameter of the nanowire the width of the resistive transition decreases. Our estimations, based on Giordano expressions,[27] showed that for sample *A* even at $T=0$ the number of quantum phase slip events should be about $\sim 10^5$ per second. That is the reason why we did not observe in our experiment any hysteresis in the current driven regime (see below). But this rate is not large enough to destroy the S behavior in the voltage driven regime. Indeed, the period of oscillations of the current for Pb is about $10^{-9}$ s at $V \sim V_1$ and $T=0$. This means that only at temperatures close to $T_c$ the fluctuating PSC's will interrupt the internal temporal oscillations in the order parameter and ruin the S behavior. In other words the system can be in a state with $j > j_{c1}$ only during a time which is less than the time between two phase slips. This results in the coincidence of the *I-V* characteristics both in the current and the voltage driven regimes at temperatures close to $T_c$ (see Fig. 3 in Ref. 25). We would like also to mention the small difference in the critical temperatures which implies that all our samples have practically the same value of the superconducting gap.

As can be seen in Fig. 12, the wider the transition width in $R(T)$, the smaller the magnetic field where the resistance starts to increase with increasing magnetic field.

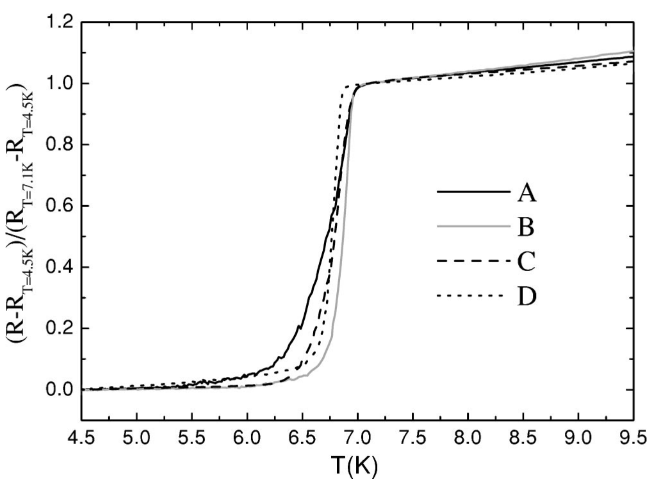


FIG. 11. Resistive transition of our different samples. Note that for the narrowest sample (A) the width of the transition is widest and for the widest sample (D) it is narrowest.

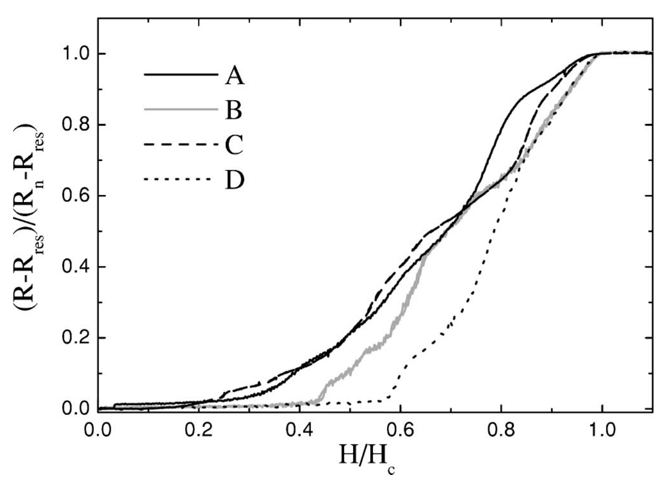


FIG. 12. Magnetoresistance of our different samples at 4.3 K.

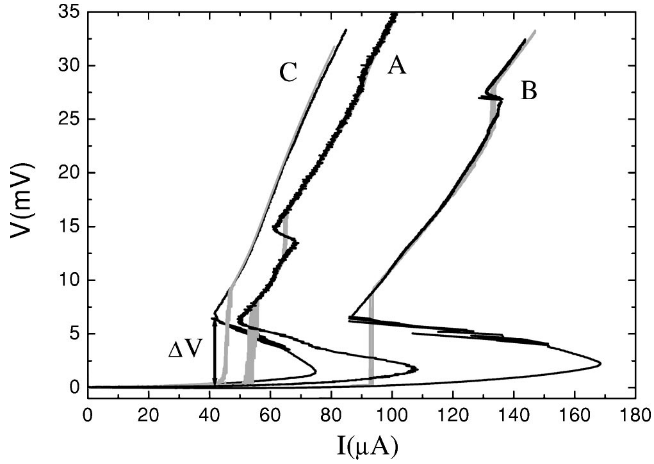


FIG. 13. Current-voltage characteristics of samples A ($T=4.3$ K), B ($T=4.37$ K), and C ($T=4.2$ K) in the current (grey curves) and voltage (black curves) driven regimes. The low current residual voltage was subtracted from the experimental data.

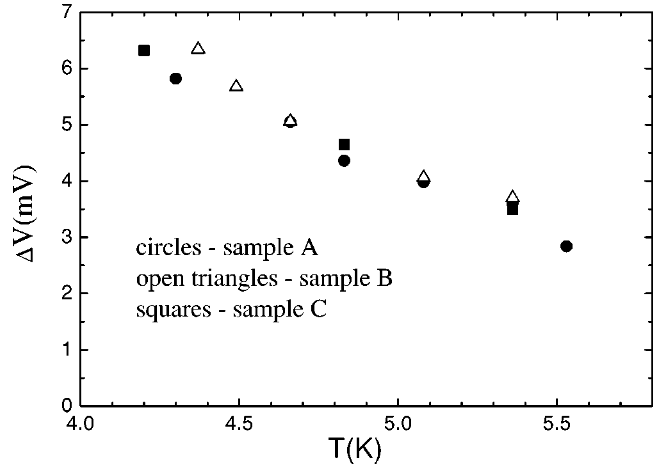


FIG. 14. Jump in the voltage $\Delta V$ [see Figs. 5(a) and 13] for samples A–C at different temperatures.

Of course, our nanowires are not free from imperfections. Although our fabrication technique produce samples which are quite regular (see Ref. 25 for the scanning electron microscopy picture), we may distinguish two kinds of imperfections. First, there can be deviations from an ideal cylindrical shape. Indeed, although the nanopores of the membrane in which the nanowires are grown are designed to be as regular as possible, there may exist a smooth variation in diameter along the length of the nanowire, which we estimated to be not more than 10%. However, in addition to these smooth variations, it may happen that a defect in the membrane leads to a constriction in the measured nanowire. In this case, the diameter can be locally reduced quite importantly and this results in a lower critical current and a larger critical magnetic field. This is in fact what we observed in sample C. In addition to such shape imperfections, we note also that the parameters of our nanowires strongly vary from sample to sample—see Table I (we estimated the coherence length using the expression $H_c=2.9\Phi_0/(\pi\xi d)$ ($\xi_1$) and $H_c=\Phi_0/(2\pi\xi^2)$ ($\xi_2$) for the critical field). The reason for this difference in resistivity is related to the second kind of imperfections, which is structural disorder that is formed during the electrodeposition of these nanowires inside the nanopores. Indeed, as shown in Ref. 29, these nanowires are polycrystalline and inevitably contain structural defects such as dislocations, twins, etc. These two kinds of imperfections results in differences in the *I*-*V* characteristics (Fig. 13). For example for samples A and B we observed two jumps in the voltage which we explain by the appearance of two successive phase slip centers in the nanowire but for sample C we found only one jump in the voltage. Indeed, due to the presence of a constriction in this sample, heating may drastically affect the behavior of this sample, precipitating the return to the normal state. Unfortunately sample D was broken during the measurements and we could not measure its *I*-*V* characteristic.

We found that jumps in the voltage $\Delta V$ are practically the same for all three samples (see Figs. 13 and 14). It agrees with the theory presented in Sec. II. Indeed all the three samples have almost the same $T_c$ and hence the same superconducting gap. It is also natural to suppose that the time $\tau_E$ is almost the same for all our samples. We can expect that the parameters $\gamma$ and $u$ are the same for all our samples and consequently the relaxation time $\tau_{|\psi|}$ and the ratio $\Lambda_Q/\xi$ are the same. This automatically leads to the invariance of $\Delta V$ on $\rho_n$. From the results of Sec. II B it follows that the voltage $V_2$ decreases with increasing (in $\xi$) nanowire length (see Fig. 6). For our shortest sample A: $L\simeq(600\text{–}1400)\xi$ (see Table I). This means that in our nanowires the voltage $V_2$ already reaches the minimal value $\Delta V$ and hence $V_2$ is independent of $\rho_n$ for our nanowires. The voltage $V_1$ depends not only on the length of the nanowire but also on the relaxation time of the order parameter during the transition period $T_0$ [see Eq. (9)] and hence the voltage $V_2$ may reach $\Delta V$ at shorter lengths than $V_1$ if the time $T_0$ is large enough (in Fig. 6 the opposite situation is presented—at first $V_1$ reaches the saturated value).

If the current density is uniformly distributed over the cross section of the sample then the oscillations of the order parameter will be in phase along the cross section, and in this case $\Delta V$ will not depend on the size of the sample. It is a direct consequence of the fact that the time-averaged chemical potential of the superconducting electrons should be equal and constant on both sides of the phase slip center or phase slip line/surface.

Our results show that if one uses the Skocpol-Beasley-Tinkham (SBT) (Ref. 8) model for the estimation of $\Lambda_Q$ one should be very careful. Indeed, those authors replaced the normal current density in the PS center by the expression $I_n(0)=(I-\beta I_{c1})$. As a result the derivative $dI_n(0)/dI=1$ becomes current independent. But in general the time derivative may be larger than unity [see Figs. 1(a) and 1(b)]. Second, if the length of the sample is comparable with $\Lambda_Q$ we should replace $2\Lambda_Q$ by $2\Lambda_Q\tanh(L/2\Lambda_Q)$ [see Eq. (7)]. As a result the differential resistance in samples with $L\sim\Lambda_Q$ may be larger, in general, than the normal one.[30] This case corresponds to our samples ($R_{dif}\simeq320\ \Omega$, $R_{dif}\simeq351\ \Omega$, and $R_{dif}\simeq456\ \Omega$ for samples A, B, and C respectively after the first jump in voltage). But from the SBT model follows that

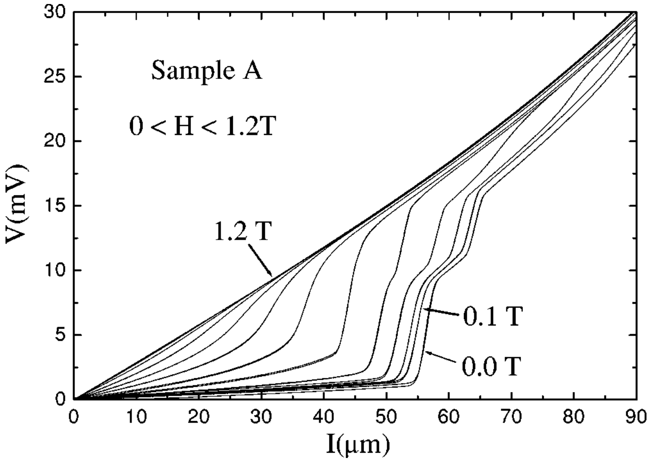


FIG. 15. Current-voltage characteristics of sample A ($T = 4.3$ K) in the current driven regime in the presence of a parallel magnetic field. The magnetic field increases from right to left with steps of $0.1T$.

$R_{dif} \leqslant R_n$ (the equality sign holds for samples with $L \lesssim \Lambda_Q$).

Unfortunately we do not know the actual dependence $I_n(0)(I)$ for our samples. If we use the values obtained from the SBT model ($\Lambda_Q = 12.6$, 48.5, and 30.3 $\mu$m for samples A, B and C, respectively) which is qualitatively understandable for longer samples, we have a too small number of PSC (compare samples A and C). And in this case, the question arises why $\Lambda_Q$ changes so much. Probably, the SBT model gives us only the correct order of magnitude for $\Lambda_Q$ which is only useful as an estimate.

Finally, we present our results on the influence of an applied magnetic field on the *I-V* characteristics. We limit ourselves to data for sample A in the current driven regime (see Fig. 15). These results already support our theoretical predictions of Sec. II B. It is evident that the lower critical current density decreases with increasing applied magnetic field. For some range of *H* values only one jump in the voltage exists. We explain it by an increase of $\Lambda_Q$ at relatively large magnetic fields and hence there is a lack in space for the coexistence of two phase slip centers in the nanowire. At high magnetic fields the order parameter is strongly suppressed by *H* and the effect of quantum phase-slip fluctuations becomes more pronounced. This is the reason for a smoothing of the *I-V* characteristics at high magnetic fields.

## IV. DISCUSSION

In conventional superconductor near $T_c$ the *N-S* boundary conditions (in the sense mentioned in Sec. II A) are valid.[17] For $T \to 0$ the bridge boundary conditions are more applicable due to Andreev reflections at the border between the normal metal and the superconductor.[17,31] At intermediate temperatures we expect a mixture of the *N-S* and bridge boundary conditions. It means that part of the voltage will drop at the boundaries and part in the sample. The situation in our experiment is even more complicated because in our two-point measurements there is also a voltage drop at the contacts. Probably, this will not allow us to observe the oscillations of the current in the voltage driven regime. Another reason is that our samples are very long compared to $\xi$ (even for our 22-$\mu$m sample $L \simeq 1000\xi$) and consequently the amplitude of the oscillations is very small. But nevertheless our calculations in both limiting cases of bridge and *N-S* boundary conditions predicts an S shape of the *I-V* characteristic.

We found that the type of boundary conditions are not so important for the process of nucleation of phase slip centers if the length of the sample is much larger than $\Lambda_Q$. However, the difference in the process of the conversion of superconducting electrons to normal ones and vice versa at the *N-S* boundary at various temperatures plays a crucial role for the creation of phase slip centers in shorter nanowires. It turned out that for similar parameters (nanowire's length, coherence length, superconducting gap, $\tau_E$) phase slip centers appear in the superconducting wire at smaller currents for the case of the bridge geometry boundary conditions.

All our theoretical results are strictly speaking only valid near $T_c$, which is the temperature region where Eqs. (1) and (2) are quantitatively correct. Nevertheless, experimental results supports our prediction on the competition of two relaxation times in the creation of a phase slip center even far from $T_c$. Indeed, an applied parallel magnetic field decreases the lower critical current (compare Figs. 15 and 4). Jumps in the voltage $\Delta V$ turned out to be almost an independent function of the disorder (of the resistance of the sample) as it follows from theory. Unfortunately, it is rather difficult to check the dependence of $j_{c1}$ on the length of the nanowire using our technique because every preparation of a new sample leads to a different level of disorder and hence different values for $\xi$ and $\Lambda_Q$.

Therefore, we expect only quantitative differences in the dependence of $j_{c1}(T,H)$, $j_{c2}(T,H)$, as compared with our theoretical results. Some qualitative differences (see Ref. 32) in the dynamic of the order parameter at the phase slip center or the creation of charge imbalance waves[33] cannot affect the main properties of our theoretical results because it does not influence the existence of the two different critical currents: $j_{c1}$ and $j_{c2}$. This may lead to quantitative differences in the dependence of $\tau_\phi$ and $\tau_{|\psi|}$ on the microscopic parameters of the superconductor.

Finally, we would like to discuss other mechanisms which, for our geometry, may lead to an S behavior of the *I-V* in the $V=$const regime. The first is heating.[34] In order to explain the double S structure for our samples A and B by this mechanism we have to assume that heat dissipation and heat evacuation have a very complicated and non trivial dependence on temperature. We do not know any mechanisms which can lead to such a dependence in our case. In addition, we did not observe any hysteresis in the current driven regime which is an inevitable property of that mechanism if the *I-V* characteristic would have a S shape in the voltage driven regime. For these reasons we believe that heating is not responsible for the observed behavior.

Second, if the nanowire contains a normal region the $I$-$V$ characteristic will exhibit an S behavior due to multiple Andreev reflections in the SNS structure.[35] We do not have any indication of this process from our $R(T)$ and $R(H)$ measurements which shows that our samples are quite homogeneous. Furthermore the structure would occur at voltages much smaller than $\Delta/e$. In our case the S behavior is seen for $V > \Delta/e$; see Fig. 13 [for Pb, $\Delta(0) \simeq 1.4$ meV]. Andreev reflection on the $N$-$S$ boundaries may give rise to a zigzag shape of the $I$-$V$ in the $V$=const regime[36] but this effect is negligible for our samples with $L \gg \xi_0$.

**ACKNOWLEDGMENTS**

This work was supported by the Belgian Science Policy through the Interuniversity Attraction Pole Program PAI (P5/1/1), GOA (University of Antwerp), ESF on ''Vortex matter,'' the Flemish Science Foundation (FWO-Vl) and the ''Communauté Française de Belgique'' through the program ''Actions de Recherches Concertées.'' D.V. was supported by DWTC to promote S & T collaboration between Central and Eastern Europe. We thank the POLY lab. at UCL for the polymer membranes. S. Mi. thanks the FNRS Belgium for financial support.

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