# Dynamic transitions between metastable states in a superconducting ring

D. Y. Vodolazov and F. M. Peeters*
*Departement Natuurkunde, Universiteit Antwerpen (UIA), Universiteitsplein 1, B-2610 Antwerpen, Belgium*
(Received 7 June 2002; published 29 August 2002)

Applying the time-dependent Ginzburg-Landau equations, transitions between metastable states of a superconducting ring are investigated in the presence of an external magnetic field. It is shown that if the ring exhibits several metastable states at a particular magnetic field, the transition from one metastable state to another one is governed by *both* the relaxation time of the absolute value of the order parameter $\tau_{|\psi|}$ and the relaxation time of the phase of the order parameter $\tau_{\phi}$. We found that the larger the ratio $\tau_{|\psi|}/\tau_{\phi}$, the closer the final state will be to the absolute minimum of the free energy, i.e., the thermodynamic equilibrium. The transition to the final state occurs through a subsequent set of *single* phase slips at a particular point along the ring.

DOI: 10.1103/PhysRevB.66.054537 PACS number(s): 74.60.Ec, 74.20.De, 73.23.−b

## I. INTRODUCTION

The fast development of experimental techniques makes possible the study of physical properties of samples with sizes of about several nanometers. The interest in such small objects is due to the appearance of new effects if the size of the system is comparable to some characteristic length. In the case of superconductors it means that the sample should have one size at least of order of the coherence length $\xi$ (such superconductors are called "mesoscopic" superconductors). For example, only for hollow cylinders or rings of radius $\sim\xi$ it is possible to observe the famous Little-Parks oscillations.[1]

The majority of the research in this area has been limited to the study of static or quasistatic properties of mesoscopic superconducting rings, disks, and other geometries.[2] It is surprising that the dynamics in such systems was practically not studied. However, it turns out that the dynamics is very important for systems that exhibit a series of metastable states and that may be brought far from thermal equilibrium. For such systems the fundamental problem is to determine the final state to which the system will transit (see for example Ref. 3).

In the present work, we present a detailed study of the dynamic transitions between different states in a mesoscopic superconducting ring. This system is a typical example of the above-mentioned systems where a set of metastable states exist. We will show that the final state depends crucially on the ratio between the relaxation time of the absolute value of the order parameter and the time of change of the phase of the order parameter. Our theoretical results explain the recent magnetization results of Pedersen *et al.*[4] in thin and narrow superconducting Al rings.

To solve this problem we present a numerical study of the time-dependent Ginzburg-Landau (TDGL) equations. Therefore, our results will also be applicable to other systems (for example, liquid helium) where the dynamics are described by these or similar equations. In our approach we neglect the self-field of the ring (which is valid if the width and thickness of the ring are less than $\lambda$, the London penetration length) and hence the distribution of the magnetic field and the vector potential are known functions. The time-dependent GL equations in this case are

$$u\left(\frac{\partial\psi}{\partial t}+i\varphi\psi\right)=(\nabla-\mathrm{i}\mathbf{A})^2\psi+\psi(1-|\psi|^2)+\chi, \quad (1a)$$

$$\Delta\varphi=\mathrm{div}\{\mathrm{Im}[\psi^*(\nabla-\mathrm{i}\mathbf{A})\psi]\}, \quad (1b)$$

where $\psi=|\psi|e^{i\phi}$ is the order parameter, the vector potential $A$ is scaled in units $\Phi_0/(2\pi\xi)$ (where $\Phi_0$ is the quantum of magnetic flux), and the coordinates are in units of the coherence length $\xi(T)$. In these units the magnetic field is scaled by $H_{c2}$ and the current density, $j$, by $j_0=c\Phi_0/8\pi^2\lambda^2\xi$. Time is scaled in units of the Ginzburg-Landau relaxation time $\tau_{\mathrm{GL}}=4\pi\sigma_n\lambda^2/c^2$, the electrostatic potential $\varphi$ is in units of $c\Phi_0/8\pi^2\xi\lambda\sigma_n$ ($\sigma_n$ is the normal-state conductivity), and $u$ is a relaxation constant. In our numerical calculations we used the new variable $U=\exp(-i\int\mathbf{A}\,d\mathbf{r})$, which guarantees gauge invariance of the vector potential on the grid. We also introduced small white noise $\chi$ in our system, the size of which is much smaller than the barrier height between the metastable states.

We consider $u$ as an adjustable parameter, which is a measure of the different relaxation times (for example, the relaxation time of the absolute value of the order parameter) in the superconductor. This approach is motivated by the following observations. From an analysis of the general microscopic equations, which are based on the BCS model, the relaxation constant $u$ was determined in two limiting cases: $u=12$ for dirty gapless superconductors[5] and $u=5.79$ for superconductors with weak depairing.[6,7] However, this microscopic theory is built on several assumptions—for example, the electron-electron interaction is of the BCS form, which influences the exact value of $u$. On the other hand, the stationary and time-dependent Ginzburg-Landau equations, Eqs. (1a) and (1b), are in some sense more general, and their general form does not depend on the specific microscopic model (but the value of the parameters, of course, are determined by the microscopic theories).[8]

The paper is organized as follows. In Sec. II we present our numerical results for the solution of Eqs. (1a) and (1b) for the strictly one-dimensional ring. In Sec. III we explain the obtained results in terms of different time scales of the

 

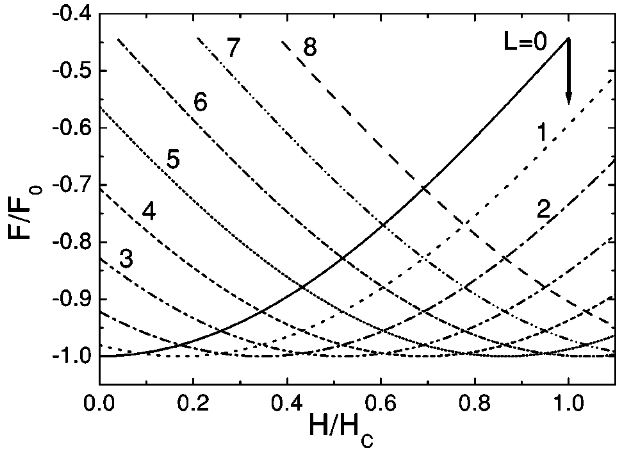


FIG. 1. Dependence of the free energy of the one-dimensional ring (with $R=10\xi$) on the applied magnetic field for different vorticity $L$.

mesoscopic superconductor. The effect of the finite width of the ring is studied in Sec. IV, where we also discuss the influence of nonzero temperature and finite $\lambda$.

## II. SUPERCONDUCTING RING OUT OF EQUILIBRIUM

If the width of the ring is much smaller than $\xi$ and $\lambda_\perp=\lambda^2/d$ ($d$ is the film thickness) it is possible to consider the ring, of radius $R$, as a one-dimensional object. At first we limit ourselves to the solution of the strictly one-dimensional equations (1a) and (1b) that model the dynamics of a ring with small width. In our approach the vector potential is equal to $A=HR/2$, where $H$ is the applied magnetic field.

As shown in Refs. 9 and 10 the transition of the superconducting ring from a state with vorticity $L=\oint\nabla\phi\, ds/2\pi$ (which in general can be a metastable state) to a state with a different vorticity occurs when the absolute value of the gauge-invariant momentum $\mathbf{p}=\boldsymbol{\nabla}\phi-\mathbf{A}$ reaches the critical value

$$p_c=\frac{1}{\sqrt{3}}\sqrt{1+\frac{1}{2R^2}}. \tag{2}$$

Let us, for simplicity, consider that the magnetic field is increased from zero (with initial vorticity of the ring $L=0$) to the critical $H_c$, where $p$ becomes equal to $p_c$ (for $L=0$ we have $H_c=2p_c/R$). In this case $\mathbf{p}=-\mathbf{A}$ and we can write Eq. (2) as

$$\Phi_c/\Phi_0=\frac{R}{\sqrt{3}}\sqrt{1+\frac{1}{2R^2}}. \tag{3}$$

For this value of the flux the thermodynamical equilibrium state becomes $L_{\mathrm{eq}}=\mathrm{Int}(\Phi_c/\Phi_0)$. For example, if $R=10$ we find $L_{\mathrm{eq}}=6$ (see Fig. 1). The fundamental question we want to answer is: What will be the actual value of the vorticity of the final state? Will it be the thermodynamic equilibrium state or a metastable one? This answer will be obtained from a numerical solution of the time-dependent Ginzburg-Landau equations.

For our numerical calculations we considered two different rings with radii $R=10$ and $R=15$. These values were chosen such that the final state exhibits several metastable states and, in principle, transitions can occur with jumps in $L$ that are larger than unity. For $R=10$, $\Delta L$ may attain values between 1 and 6, and for $R=15$ it may range from 1 to 9. Smaller rings have a smaller range of possible $\Delta L$ values. We found that larger rings than $R=15$ did not lead to new effects and are therefore not considered here. In our numerical simulations we increased $H$ gradually from zero to a value $H_c+\Delta H$ (we took $\Delta H=0.036H_c$ for $R=10$ and $\Delta H=0.012H_c$ for $R=15$) during a time interval $\Delta t$, after which the magnetic field was kept constant. The magnetic field $\Delta H$ and time $\Delta t$ ranges were chosen sufficiently large in order to speed up the initial time for the nucleation of the phase slip process, but still sufficiently small in order to model real experimental situations in which $H$ is increased during a time much larger than $u\tau_{\mathrm{GL}}$ [for example, for Al $\tau_{\mathrm{GL}}(T=0)\simeq10^{-11}$ s]. A change of $\Delta H$ and $\Delta t$, within realistic boundaries, did not have an influence on our final results.

From our detailed numerical analysis, we found that the vorticity $L$ of the final state of the ring depends on the value of $u$ and is not necessarily equal to $L_{\mathrm{eq}}$. The larger the $u$ the larger the vorticity after the transition. The final state is reached in the following manner. When the magnetic field increases the order parameter decreases [see inset of Fig. 2(a)]. First, in a single point of the ring a local suppression, in comparison with other points, of the order parameter occurs, which deepens gradually with time during the initial part of the development of the instability. This time scale is taken as the time at which the value of the order parameter decreases from 1 to 0.4 in its minimal point. When the order parameter reaches the value $\sim0.4$ in its minimal point the process speeds up considerably and the order parameter starts to oscillate in time at this point along the ring [see Fig. 2(a)]. At the same time the oscillatory behavior of the superconducting $j_s=\mathrm{Re}[\psi^*(-i\boldsymbol{\nabla}-\mathbf{A})\psi]$ [Fig. 2(b)] and the normal $\boldsymbol{j}_n=-\boldsymbol{\nabla}\varphi$ [Fig. 2(c)] current density is found at the point where the minimal value of the order parameter is found. At other places in the ring such oscillatory behavior is strongly damped (see, e.g., the thin curves in Fig. 2). After some time the system evolves to a new stable state, which in the situation of Fig. 2 is $L=3$ for $R=10$ and $L=5$ for $R=15$.

In Fig. 3 the dependence on $u$ of the dynamics of $|\psi|$ in the minimal point is illustrated. Note that with increasing value of $u$ the number of phase slips (or equivalently the number of oscillations of the order parameter at its minimal point) increases and hence $\Delta L$ also increases. Another important property is that the amplitude of those oscillations decreases with increasing $u$.

## III. TIME SCALES

The above results lead us to conclude that in a superconducting ring there are two characteristic time scales. First, there is the relaxation time of the absolute value of the order parameter $\tau_{|\psi|}$. The second time scale is determined by the time between the phase slips (PS's), i.e., the period of oscillation of the value of the order parameter at its minimal point (see Fig. 3). Below we show that the latter time is directly

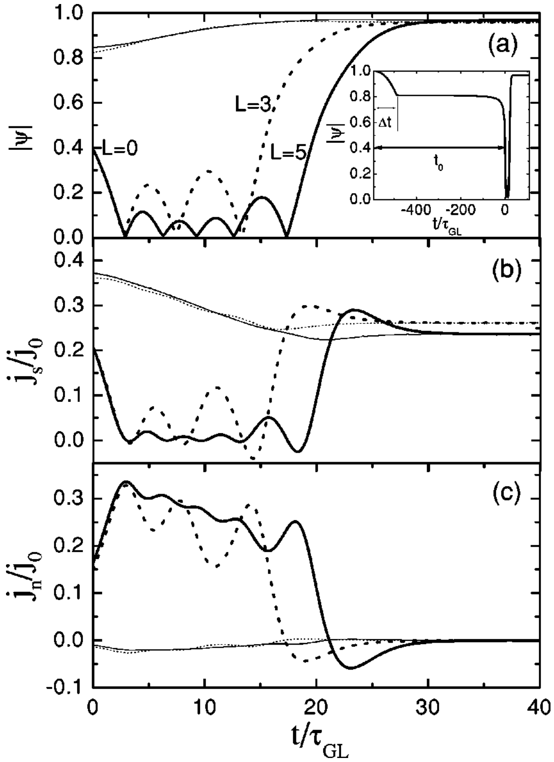


FIG. 2. Dependence of (a) the order parameter, (b) the superconducting $j_s$, and (c) the normal $j_n$ current density at the point where the minimal (thick curves) and the maximal (thin curves) values of the order parameter is reached as function of time for $u=3$, $R=10$ (dotted curves) and $R=15$ (solid curves). For both rings we took $\Delta t=100$, the time interval over which the magnetic field is increased to its value $H_c$ [inset of (a)].

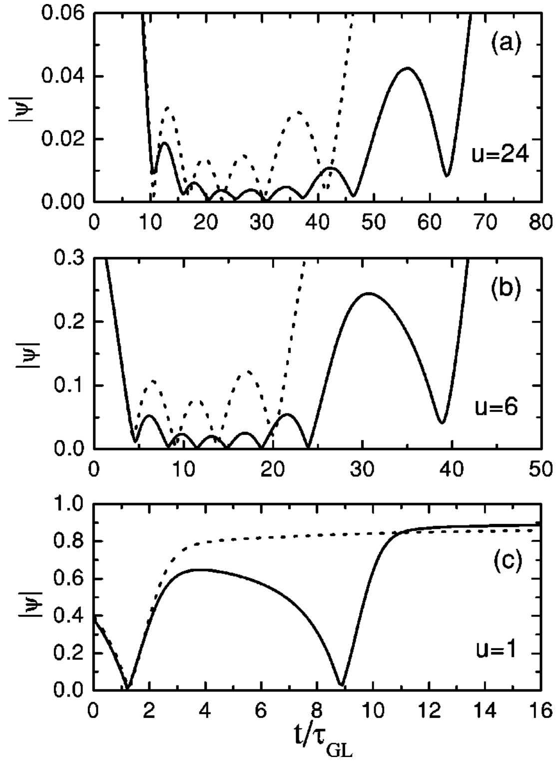


FIG. 3. The dynamics of the order parameter in its minimal point for different values of the parameter $u$ and for two values of the radius $R=10\xi$ (dotted curves) and $R=15\xi$ (solid curves). The zero of time is taken at the moment when the value of the order parameter becomes equal to 0.4 in its minimal point. Nonzero values of $|\psi|$ near the moment of the phase slip are connected to the finite coordinate and time step used in the numerical calculations.

proportional to the time of change of phase of the order parameter $\tau_\phi$ and is connected to the relaxation time of the charge imbalance in the system.

In Fig. 4 the distribution of the absolute value of the order parameter $\psi$ and the gauge-invariant momentum $p$ are shown near the point where the first phase slip occurs for a ring with radius $R=10$ and $u=3$ at different times: just before and after the first PS which occurs at $t\simeq 2.7\tau_{\mathrm{GL}}$. Before the moment of the phase slip the order parameter decreases while after the PS it increases. In order to understand this different behavior let us rewrite Eq. (1a) separately for the absolute value $|\psi|$ and the phase $\phi$ of the order parameter

$$u\frac{\partial|\psi|}{\partial t}=\frac{\partial^2|\psi|}{\partial s^2}+|\psi|(1-|\psi|^2-p^2), \tag{4a}$$

$$\frac{\partial\phi}{\partial t}=-\varphi-\frac{1}{u|\psi|^2}\frac{\partial j_n}{\partial s}. \tag{4b}$$

Here $s$ is the arc coordinate along the ring and we used the condition $\mathrm{div}(j_s+j_n)=0$. It is obvious from Eq. (4a) that if the right-hand side (RHS) of Eq. (4a) is negative, $|\psi|$ decreases in time, and if the RHS is positive, $|\psi|$ will increase in time. Because the second derivative of $|\psi|$ is always positive [at least near the phase-slip center—see Fig. 4(a)] the different time dependence of $|\psi|$ is governed by the term $-p^2|\psi|$. From Fig. 4(b) it is clear that after the phase slip the value of $p$ is less than before this moment, with practically the same distribution of $|\psi|$. It is this fact that is responsible for an increase of the order parameter just after the

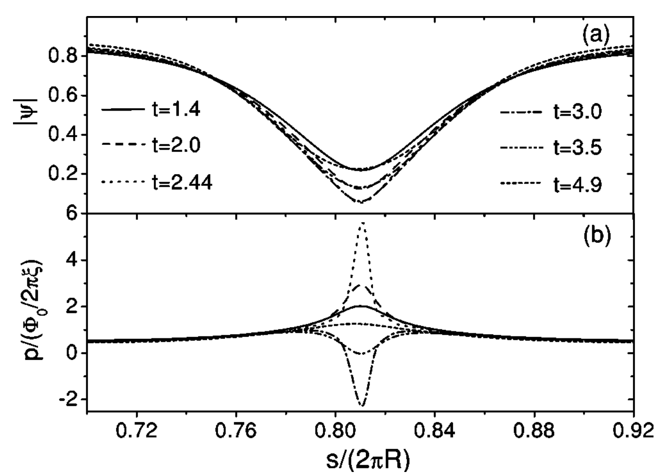


FIG. 4. Distribution of (a) the absolute value of the order parameter and (b) gauge-invariant momentum near the phase-slip center at different moments of time for a ring of $R=10\xi$ and with $u=3$. The phase slip occurs at $t\simeq 2.7\tau_{\mathrm{GL}}$.

moment of the phase slip. But at some moment of time the momentum $p$ can become sufficiently large, making the RHS of Eq. (4a) negative, and as a consequence $|\psi|$ starts to decrease.

Based on our numerical calculations we may state that for every value of the order parameter in the minimal point, there exists a critical value of the momentum $p_c^{\min}$ such that if the value of the momentum in this point is less than $p_c^{\min}$, the order parameter will increase in time. In the opposite case the order parameter decreases, which leads ultimately to the phase-slip process. Therefore, when after a phase slip $p$ increases fast enough such that at some moment the condition $p^{\min}>p_c^{\min}$ is fulfilled, the order parameter will start to decrease, which leads then to a new phase-slip process.

From our results we can conclude that the change of $p$ (or phase of the order parameter) with time and of $|\psi|$ with time has a different dependence on $u$. Indeed from Figs. 2 and 3 it follows that with increasing $u$ the time relaxation of $|\psi|$ becomes larger with respect to the relaxation time for $p$ (for example, the amplitude of oscillations of $|\psi|$ is decreased). Moreover, the relaxation time for $p$ depends not only on $u$ but also on the history of the system: the larger the number of phase slips that have occurred in the system, the longer the time between the next two subsequent phase slips. In order to understand such behavior we turn to Eq. (4b).

Numerical analysis shows that the second term in the RHS of Eq. (4b) is only important for some length near the phase-slip center. This length is nothing else than the length over which, the electric field penetrates into the superconductor. We checked this directly by solving Eqs. (1a) and (1b) for this situation. But this length is the decay length $\lambda_Q$ of the charge imbalance in the superconductor (see, for example, Refs. 11 and 12). We numerically found that $\lambda_Q$ varies with $u$ (in the range $u=1-100$) as $\lambda_Q \sim u^{-0.27}$. It means that the relaxation time of the charge imbalance is $\tau_Q \sim \lambda_Q^2 \sim u^{-0.54}$.

Lets take the derivative $\partial/\partial s$ on both sides of Eq. (4b) and integrate it over a distance of $\lambda_Q$ near the PS center (far from it we have $j_n \sim 0$). We obtain the Josephson relation (in dimensionless units)

$$\frac{d\Delta\phi}{dt} = V \sim j_n(0)\lambda_Q, \tag{5}$$

where $\Delta\phi = \phi(+\lambda_Q) - \phi(-\lambda_Q)$ is the phase difference over the phase-slip center, which leads to the voltage $V = -[\varphi(+\lambda_Q) - \varphi(-\lambda_Q)]$ and where $j_n(0)$ is the normal current (or electric field in our units) in the point of the phase slip. From Eq. (5) it follows immediately that the relaxation time for the phase of the order parameter near the phase slip center is

$$\tau_\phi \sim \frac{1}{\lambda_Q \langle |j_n(0)| \rangle}, \tag{6}$$

where $\langle \cdot \rangle$ means averaging over time between two consecutive phase slips. This result allows us to qualitatively explain our numerical results. Indeed, from Fig. 2(c) it is apparent that $\langle |j_n(0)| \rangle$ decreases after each PS and as a result it leads to an increase of $\tau_\phi$. With increasing $u$, $\lambda_Q$ decreases and $\tau_\phi$ increases. Fitting our data (see Fig. 5) leads to the dependencies $\tau_\phi \sim u^{0.21}$ and $\tau_\phi \sim u^{0.23}$ for rings with radius $R=10$ and $R=15$, respectively (here $\tau_\phi$ was defined as the time between the first and the second phase slip). This is close to the expected dependencies that follow from Eq. (6) and from the above dependence of $\lambda_Q(u)$ (quantitative differences follow from uncertainty in our finding $\tau_\phi$ and $\lambda_Q$). Besides, Eq. (6) allows us to explain the decrease of $\tau_\phi$ with increasing radius of the ring (see inset in Fig. 5). Namely, during the time between two phase slips the gauge-invariant momentum decreases as $\Delta p \sim 1/R$ (because $\nabla\phi$ increases as $\sim 2\pi/2\pi R$) in the system. Far from the phase-slip center the total current practically is equal to $j_s \sim p$. Because $\mathrm{div}(j_s + j_n) = 0$ in the ring we see directly that during this time the normal current density in the point of the phase slip also decreases as $\sim 1/R$. Taking into account Eq. (6) we can conclude that $\tau_\phi$ should vary as $\sim 1/R$ (at least for a large radius and for the first phase slip). The behavior shown in the inset of Fig. 5 is very close to such a dependence (it is interesting to note that in contrast to $\tau_\phi$ the time $\tau_{|\psi|}$ does not actually depend on $R$).

On the basis of our results we can make the following statement: When the period of oscillation (time of change of the phase of the order parameter) becomes of the order, or larger, than the relaxation time of the absolute value of the order parameter the next phase slip becomes impossible in the system.

This result can be applied to the system of Ref. 13 where a current-carrying wire was studied. The authors found a critical current $j_c = 0.335$ for $u=5.79$ and $j_c = 0.291$ for $u=12$. From our observation follows that the phase-slip solution may be realized as a stable solution when $\tau_{|\psi|} > \tau_\phi \sim 1/(\lambda_Q \langle |j_n(0)| \rangle)$. We found that $\lambda_Q \sim u^{-0.27}$, $\tau_{|\psi|} \sim u^{0.6}$,

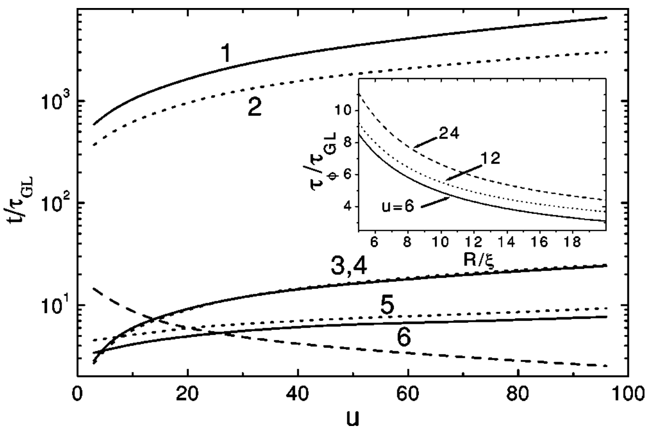


FIG. 5. Dependence of the initial nucleation time $t_0$ (curves 1,2), the relaxation time of the order parameter $\tau_{|\psi|}$ (which for definiteness we defined as the time of variation of the order parameter from 0.4 till the first PS, curves 3,4), and the relaxation time of the phase of the order parameter $\tau_\phi$ (we defined it here as the time between the first and second PS, curves 5,6) are shown as a function of the parameter $u$. Dotted (solid) curves are for a ring with radius $R=10$ (15). The dashed curve corresponds to the relaxation time of the charge imbalance (as obtained from the expression $\tau_Q = 5.79\lambda_Q^2$). In the inset, the dependency of $\tau_\phi$ on the ring radius is shown for different values of the parameter $u$.

and $\langle|j_n(0)|\rangle \sim j_{\rm ext}$, which leads to the critical current $j_c \sim u^{-0.33}$, which decreases with increasing $u$.[14]

Above we found that the time scale governing the change in the phase does not coincide with the relaxation time of the absolute value of the order parameter. This difference is essentially connected with the presence of an electrostatic potential in the system. In order to demonstrate this we performed the following numerical experiment. We neglected $\varphi$ in Eq. (1a) and found that the number of PS's is now independent of $u$ and $R$, and $\Delta L$ equals unity. Moreover, it turned out that the time scale $t_0$ is an order of magnitude larger. This clearly shows that the electrostatic potential is responsible for the appearance of a second characteristic time that results in the above-mentioned effects.

In an earlier paper[9] the question of the selection of the metastable state was already discussed for the case of a superconducting ring. However, these authors neglected the effect of the electrostatic potential and found that transitions with $\Delta L > 1$ can occur only when the magnetic field [called the induced electromoving force (emf) in Ref. 9] increases very quickly. In their case these transitions were connected to the appearance of several nodes in $\psi$ along the perimeter of the ring and in each node a single PS occurred. We reproduced those results and found such transitions also for larger values of $\Delta H$. However, simple estimates show that in order to realize such a situation in practice it is necessary to have an extremely large ramp of the magnetic field. For example, for Al mesoscopic samples with $\xi(0) = 100$ nm and $R = 10\xi$ the corresponding ramp should be about $10^3$–$10^4$ T/s. With such a ramp the induced normal currents in the ring are so large that heating effects will suppress superconductivity.

The transitions between the metastable states in the ring are not only determined by $u$ and $R$, but also the presence of defects play a crucial role. Their existence leads to a decrease of $\Delta L$. This is mainly connected to the fact that with decreasing $p_c$ (as a result of the presence of a defect) the value of $\langle|j_n(0)|\rangle$ decreases even at the moment of the first phase slip (because $\langle|j_n(0)|\rangle$ cannot be larger than $\langle|j_s(0)|\rangle \simeq p_c$) and hence $\tau_\phi$ increases. We can also explain this in terms of a decreasing degeneracy of the system. For example, if we decrease $p_c$ by a factor of 2 (e.g., by the presence of defects) it leads to a twice smaller value for $\Phi_c$ and $L_{\rm eq}$. But our numerical analysis showed that the effect of defects is not only restricted to a decrease of $p_c$ and $L_{\rm eq}$. To show this we simulated a defect by inserting an inclusion of another material with less or zero $T_c$. This is done by inserting in the RHS of Eq. (1a) an additional term $\rho(r)\psi$ where $\rho(r)$ is zero except inside the defect where $\rho(r) = \alpha < 0$. The magnetic field was increased up to $H_c$ of the ideal ring case. We found that the number of PS's was smaller as compared to the case of a ring without defects. The calculations was done for defects such that $p_c$ was decreased by less than a factor of 2 in comparison to the ideal ring case. In contrast, a similar calculation for a ring with nonuniform thickness/width showed that, even for "weak" nonuniformity (which decreases $p_c$ by less than 20%), $\Delta L$ was larger than that for the ideal ring case, and the final vorticity approaches $L_{\rm eq}$. This remarkable difference between the situation for a defect and the case of a nonuniformity may be traced back to the difference in the distribution of the order parameter: even in the absence of an external magnetic field the defect leads to a nonuniform distribution of the order parameter, which is not so for the nonuniform ring case. A more thorough study of the effect of defects will be presented elsewhere.

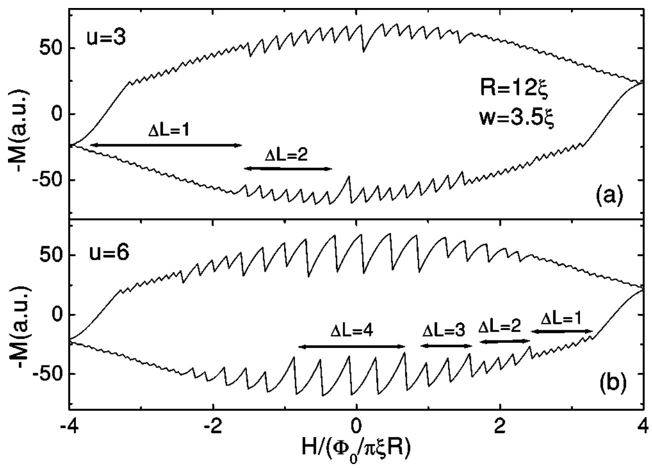


FIG. 6. Dependence of the magnetization of a finite width ring on the external magnetic field for two values of the parameter $u$. The results are shown for a sweep up and sweep down of the magnetic field.

## IV. SUPERCONDUCTING RING WITH NONZERO WIDTH

All the results above were based on a one-dimensional model that contains the essential physics of the decay and recovery of the superconducting state in a ring from a metastable state to its final state. However, even in the case when the effect of the self-field can be neglected, the finite width of the ring may still lead to important additional effects (for example, a finite critical magnetic field). In order to include the finite width of the ring into our calculation we considered the following model. We took a ring of mean radius $R = 12\xi$, width $3.5\xi$, and thickness less than $\xi$ and $\lambda$. These parameters are close to those of the ring studied experimentally in Ref. 4 and they are such that we can still neglect the self-field of the ring. The obtained magnetization curves of such a ring are shown in Fig. 6 as a function of the magnetic field and for two values of $u$. Those results were obtained from a numerical solution of the two-dimensional Ginzburg-Landau equations, Eqs. (1a) and (1b). The magnetic field was changed with steps $\Delta H$ over a time interval that is larger than the initial part of the phase slip process $t_0$ (for our parameters this procedure leads practically to an adiabatic change of the magnetic field).

From Fig. 6 we notice that the value of the vorticity jumps, $\Delta L$, depends sensitively on the parameter $u$.[15] We found that the phase slips occur in one particular place along the perimeter of the ring. However, in contrast to our previous one-dimensional case, $\Delta L$ depends also on the applied magnetic field. The reason is that for a finite width ring the number of metastable states decreases with increasing magnetic field (see Ref. 16). It means that the system cannot be moved far from equilibrium with a large superconducting current density (because the order parameter is strongly suppressed by the external field) at high magnetic field. Hence, the value of $\langle |j_n(0)| \rangle$ will be much smaller in comparison with that at low magnetic fields, and $\tau_\phi$ is larger or comparable with $\tau_{|\psi|}$ even for high values of $u$ and even for the first PS. Thus, the effect of a large magnetic field in the case of a finite width ring is similar, in some respect, to the effect of defects for a one-dimensional ring.

Our numerical results are in *qualitative* agreement with the experimental results of Pedersen *et al.* (see Fig. 2 of Ref. 4). Unfortunately, no *quantitative* comparison is possible because of the number of unknowns; e.g., the value of $p_c$ [it is necessary to have the dependence of $M(H)$ as obtained starting from zero magnetic field], the value of $\xi$ is not accurately known, and hence the ratio $R/\xi$ can therefore only be estimated. The value of both these parameters have a strong influence on the value of the vorticity jumps $\Delta L$.

Let us discuss now finite temperature and screening effects (i.e., finite $\lambda$). Nonzero temperature leads to the possibility of a perturbative appearance of phase-slip centers at $H^* < H_c$. But if the field $H^*$ does not differ too much from $H_c$, the average $\langle |j_n(0)| \rangle$ is not altered strongly. As a result, after the appearance of the first thermoactivated PS center the second phase slip may appear automatically (if the ratio $\tau_{|\psi|}/\tau_\phi$ is large enough), which leads to an avalanche-type of process. Unfortunately, it is impossible to apply the results of Ref. 17 in order to calculate the probability of this process even for the first PS. The reason is that in Ref. 17 the effect of the electrostatic potential (which plays a crucial role as was mentioned above) was neglected.

A finite $\lambda$ (and large enough width of the ring) leads to a nonuniform distribution of the momentum $p$ over the width of the ring, which can play an important role on the effects considered above. Such a finite width ring may be modeled as a strip with transport current in an external magnetic field. In Ref. 18 the condition for the entry of the first vortices in a narrow superconducting strip was studied. It turned out that the period of the vortex chain entry depends essentially on the distribution of $p$ over the width of the strip. When this distribution is uniform, the period is infinite. But even small nonuniformities in this distribution lead to a finite period. If we translate their results to our ring geometry, the period of the vortex chain must now be discrete, $2\pi R/n$ ($n=1,2,\ldots$). Under a certain condition the entry of vortices becomes possible, not through a single point along the perimeter, but through two, three, and more points (without increasing the ramp of the magnetic field). Finite $\lambda$ only increases this effect because it leads to larger nonuniformities in the dependence of $p$ over the ring width. In this situation there is competition between the process of the appearance of additional nodes along the ring perimeter and the number of PS's in these points. At high magnetic fields these effects become negligibly small (because superconductivity only exists near the edges and the ring effectively can be considered as two one-dimensional rings) and we will have the situation as discussed in the present work and recently in Ref. 16.

## V. CONCLUSION

In conclusion, we studied how an unstable superconducting state of a superconducting ring evolves in time and transit to its final state. The latter is not necessary the thermodynamic equilibrium state and may be another metastable state with a different vorticity but one that is stable in time. The transition between the different superconducting states occurs through a phase-slip center, which is a point along the ring where the superconducting amplitude decreases to zero abruptly, resulting in the change of the vorticity of the superconducting state with one unit. The waiting time, or the creation time for the first phase slip, is found to be two orders of magnitude larger than the subsequent time intervals between consecutive phase slips. The latter time is connected with the time relaxation of the charge imbalance in the superconductor and increases the closer the system nears the final state. This circumstance allows us to find this time (also as the relaxation time of the absolute value of the order parameter) from magnetic measurements of superconducting rings of large radius. Our theoretical findings are in agreement with recent experimental results of Pedersen *et al.*[4]

## ACKNOWLEDGMENTS

The work was supported by the Flemish Science Foundation (FWO-Vl), the ''Onderzoeksraad van de Universiteit Antwerpen,'' the ''Interuniversity Poles of Attraction Program—Belgian State, Prime Minister's Office—Federal Office for Scientific, Technical and Cultural Affairs,'' and the European ESF–Vortex Matter. One of us (D.Y.V.) received individual support from FWO-Vl.

*Electronic address: peeters@uia.ua.ac.be

[1] W.A. Little and R.D. Parks, Phys. Rev. Lett. **9**, 9 (1962).
[2] F.M. Peeters, V.A. Schweigert, B.J. Baelus, and P.S. Deo, Physica C **332**, 255 (2000).
[3] M.C. Cross and P.C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[4] S. Pedersen, G.R. Koford, J.C. Hollingbery, C.B. Sorensen, and P.E. Lindelof, Phys. Rev. B **64**, 104522 (2001).
[5] L.P. Gor'kov and G.M. Eliashberg, Zh. Éksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
[6] A. Schmid, Phys. Kondens. Mater. **5**, 302 (1996).
[7] L. Kramer and R.J. Watts-Tobin, Phys. Rev. Lett. **40**, 1041 (1978).
[8] For a discussion of this question see T.J. Rieger, D.J. Scalapino, and J.E. Mercereau, Phys. Rev. B **6**, 1734 (1972); A.T. Dorsey, *ibid.* **46**, 8376 (1992). For a review of different types of time-dependent Ginzburg-Landau equations see B.I. Ivlev and N.B. Kopnin, Usp. Fiz. Nauk **142**, 435 (1984) [Sov. Phys. Usp. **27**, 206 (1984)]; A.L. de Lozanne and M.R. Beasley, *Nonequilibrium Superconductivity*, edited by D.N. Langenberg and A.I. Larkin (North-Holland, Amsterdam, 1986), p. 111, and Refs. 11 and 12.
[9] M.B. Tarlie and K.R. Elder, Phys. Rev. Lett. **81**, 18 (1998).
[10] D.Y. Vodolazov and F.M. Peeters, cond-mat/0201564 (unpublished).

[11] A.M. Kadin and A.M. Goldman, *Nonequilibrium Superconductivity*, edited by D.N. Langenberg and A.I. Larkin (North-Holland, Amsterdam, 1986), p. 253.
[12] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[13] L. Kramer and A. Baratoff, Phys. Rev. Lett. **38**, 518 (1977).
[14] This result *qualitatively* explains the decreasing $j_c$ with increasing $u$ as obtained in Ref. 13 because of our subjective definition of $\tau_{|\psi|}$ and $\tau_\phi$ (see, for example, the caption of Fig. 5) no simple quantitative comparison can be made.
[15] The monotonic change of the magnetization $M$ at the switching magnetic field, i.e., $|H/(\Phi_0/\pi\xi R)|$, from increasing to decreasing behavior is connected with the conservation of vorticity $L$ of the ring in this range of magnetic fields.
[16] J. Berger, cond-mat/0206314 (unpublished).
[17] D.E. McCumber and B.I. Halperin, Phys. Rev. B **1**, 1054 (1970).
[18] I. Aranson, M. Gitterman, and B.Y. Shapiro, Phys. Rev. B **51**, 3092 (1995).