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# Domain coexistence of magnetism and superconductivity: appearance of confined vortex loops

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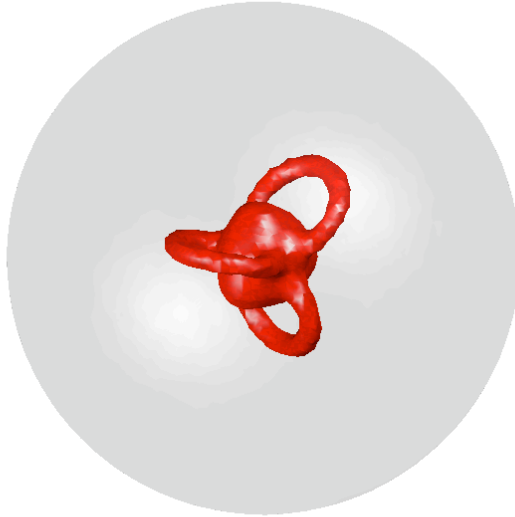
**Abstract.** A magnetic moment inside an extreme type II superconductor can have three, but not one or two, confined vortex loops near to the core. For a sub-micron superconducting particle the confined vortex loops eventually break up and reach the surface turning into external vortex pairs.

## 1. Introduction

Recently we have determined [1] the vortex state near a point like magnetic dipole  $\mu$  deep inside a superconductor such that its inhomogeneous magnetic field is the only source and sinkhole of vortex lines. Our results were obtained in the framework of the Ginzburg-Landau theory, numerically solved in three-dimensions, for a sub-micron sphere of radius  $R = 15\xi$  with the dipole in its center. We found vortex loops around the magnetic core. The vortex states are characterized by the number of these confined vortex loops (CVL) and by the broken loops. The latter corresponds to CVL that spring to the sphere's surface, split into two vortex lines, hereafter called external vortex pairs (EVP).

The CVLs and EVPs are curved vortex lines in space, truly three-dimensional spatial structures, beyond the treatment of the Ginzburg-Landau theory done in previous works [2, 3, 4] that just consider vortices as 2D "coins" inside a thin superconducting film. For increasing dipole field, and just above the Meissner phase, we find that the vortex state is made of three CVLs around the dipole axis, as illustrated in Fig. 1.

These confined vortex loops, derived through the Ginzburg-Landau theory, are found intriguingly similar to the quark confinement scenario of Quantum Chromodynamics (QCD). Quarks are confined by strong interactions[5] and form pairs or triplets so that the resulting color is neutral. The concept of color charge is beyond the present treatment which is restricted to the standard electric charge. Nearly thirty years ago 't Hooft and Mandelstam [6, 7, 8] suggested



**Figure 1.** Plot of the Cooper-pair density for a magnetic moment  $\mu/\mu_0 = 25$  in the center of a superconducting sphere of radius  $15.0\xi$ . Red and gray surfaces are parts of the same isosurface drawn at 20 % of the maximum Cooper-pair density.

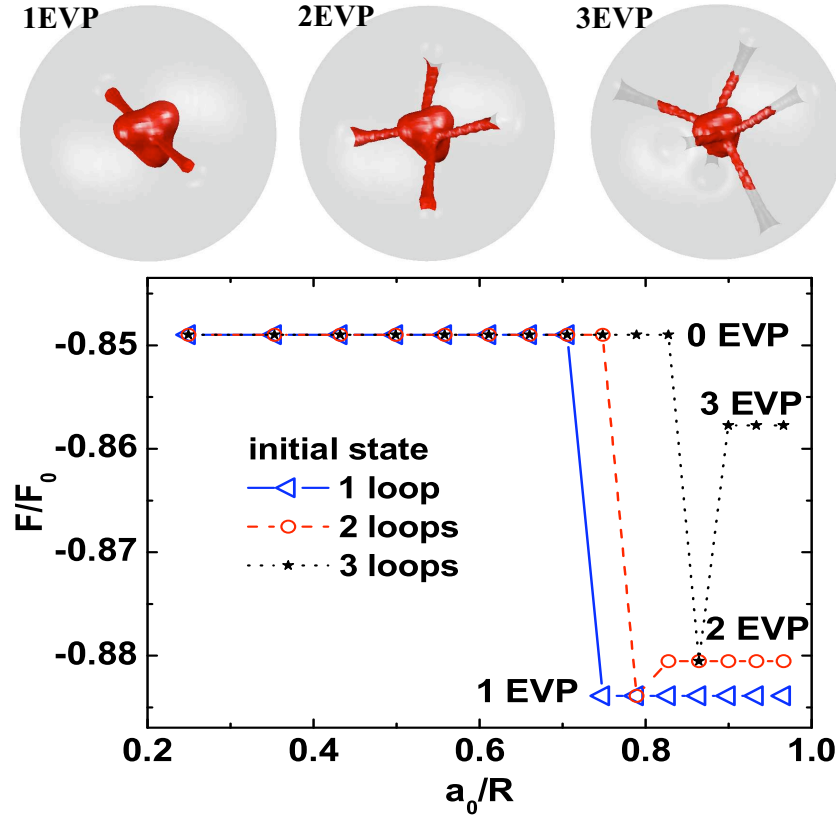
that the duality between electric charges and magnetic monopoles present in Maxwell's theory is the key to understand quark confinement. Thus instead of trying to understand the confinement of quarks (electric charges) one should look to the dual problem, the confinement of magnetic monopoles. A pair formed by a monopole and an anti-monopole inside a superconductor is realistic and experimentally feasible because it just corresponds to a tiny magnetic inclusion inside the superconductor. The magnetic streamlines coalesce into vortex tubes whose source and sinkhole are the monopole and the anti-monopole, respectively. The confined vortex loops are under pressure by the Meissner currents and attempts to separate the monopole from the anti-monopole (elongate the dipole) will result in a linear increase of energy, causing the confinement. Simply said, the superconductor, a condensate of electric charges, confines magnetic charges, in analogy to the electric charge (quark) confinement observed in our world by a condensate of magnetic monopoles.

In this work we give new arguments in support of this finding that three and not one or two loops appear connecting the monopole anti-monopole (dipole) axis. Whether this discrete number of loops has any analogy with gluons remains to be seen. We find in our Ginzburg-Landau simulations that the stability of the two loop configuration is only possible for a broken azimuthal symmetry, such as in case of a rigid lattice of magnetic moments inside a bulk superconductor [9, 10]. The magnetic field generated by a point-like dipole, or a uniformly magnetized sphere in its external region, has the following expression

$$\mathbf{B}(\mathbf{r}) = \frac{3(\boldsymbol{\mu} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \boldsymbol{\mu}}{r^3} \quad (1)$$

Since the field of Eq. (1) diverges at the dipole position, in its vicinity there is a normal region where the dipole field is bigger than  $H_{c2}$ . We can determine the frontier of this normal core imposing that the dipole field is equal to the upper critical field.

$$H_{c2} = \frac{\phi_0}{2\pi\xi^2} = \frac{3(\boldsymbol{\mu} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \boldsymbol{\mu}}{r_{c2}^3} \quad (2)$$



**Figure 2.** Free energy (arbitrary units) vs. for a dipole moment  $\mu/\mu_0 = 25$ . It shows that in case the initial loop position, given by  $a_0/R$ , is near to the edge, the onset of EVPs becomes possible.

where  $r_{c2}$  is the distance from the dipole to the  $H_{c2}$  frontier. This condition of Eq. (2) is taken where the dipole field assumes the lowest values,  $\boldsymbol{\mu} \cdot \hat{\mathbf{r}} = 0$ . We find

$$r_{c2} = \left( \frac{\mu}{\mu_0} \right)^{1/3} \xi. \quad (3)$$

In case the magnetic moment is larger than  $\mu_0 = \phi_0 \xi/2\pi$ , the normal region has radius larger than  $\xi$ . In the same way we can determine the  $H_{c1}$  frontier, where we find

$$r_{c1} = r_{c2} \left( \frac{2\kappa^2}{\ln \kappa} \right)^{1/3} \quad (4)$$

Here we consider a type II superconductor of finite size, such that the London penetration length is much larger than the sample.

Support for the stability of the three loop state is obtained by minimizing the Ginzburg-Landau free energy through the simulated annealing procedure [11, 12]. We initialize the minimization procedure upon a pre-defined order parameter,  $\Psi_N(a, \varphi)$ . This initial state depends on the following variables: the loop parameter  $a = r/\sin^2 \theta$ , the azimuthal angle around the dipole axis,  $\varphi$ , and the number of zeros of this wave function,  $N$ . The variable  $a$  describes the magnetic field streamlines around the dipole. The condition  $\Psi_N(a_0, \varphi_0) = 0$  holds for a single loop position,  $a = a_0$ , and for a special azimuthal configuration determined by the condition  $\exp(iN\varphi_0) = 1$ . Hence this initial state contains  $N$  symmetrically arranged CVLs along the magnetic field streamlines around the dipole. Fig. 2 shows the free energy value found at the end of the minimization procedure according to the initial vortex position,  $a_0/R$ . Three distinct initial states were taken, i.e.,  $N = 1, 2$  and 3 loops. One expects that the numerical procedure

should lead to a single state, a three loop final state, regardless of the number of loops present in the initial state. Indeed this is true, but only for a small  $a_0/R$  ratio, according to Fig. 2. For a large  $a_0/R$  ratio several final states are possible. The proximity to the boundaries explains this puzzling fact that for  $a_0/R > 0.73$  the free energy minimum depends on the initial state, as shown in Fig. 2. The proximity of  $N$  loops to the surface of the sphere causes the onset of EVPs during the annealing procedure. In fact for the three highest values of  $a_0/R$ , the number of EVPs is exactly the number of loops in the initial state ( $N$ ). This striking correlation leaves no doubt that an initial state with a CVL very near to the edge most easily turns into an EVP during the minimization procedure, fact that is not possible for a initial state with a CVL away from the edge. Thus for a finite superconductor the vortex state has to be classified according to the number of CVLs and of EVPs, an important difference to previous 2D studies [2, 3, 4]. Regardless of the number of EVPs all the vortex states of Fig. 2 display three embryonic CVLs very near to the  $H_{c2}$  core.

## 2. Conclusion

In conclusion, the onset of three loops from the  $H_{c2}$  core is energetically favorable over one or two vortex loops, regardless of the boundaries. This problem - mesoscopic superconductor with a magnetic inclusion in its center- can be useful to understand the quark confinement problem, according to the 't Hooft and Mandelstam conjecture.

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- [1] Doria M M, de C Romaguera A R, Milošević M V and Peeters F M 2007 *Europhys. Lett.* **79** 47006
- [2] Milošević M V and Peeters F M 2003 *Phys. Rev. B* **68** 024509
- [3] Milošević M V and Peeters F M 2005 *Phys. Rev. Lett.* **94** 227001
- [4] Milošević M V and Peeters F M 2004 *Phys. Rev. Lett.* **93** 267006
- [5] Halzen F and Martin A 1984 *Quarks & Leptons: An Introductory Course in Modern Particle Physics* (John Wiley & Sons)
- [6] 't Hooft G 1978 *Nucl. Phys.* **B138** 1
- [7] 't Hooft G 1981 *Nucl. Phys.* **B138** 455
- [8] Madelstam S 1976 *Phys. Rep. C* **23** 145
- [9] Doria M M 2004 *Physica C* **408-410** 466–469
- [10] Doria M M 2004 *Physica C* **404** 145
- [11] Doria M M, Gubernatis J E and Rainer D 1990 *Phys. Rev. B* **41** 6335
- [12] Romaguera A R d C, Doria M M and Peeters F M 2007 *Phys. Rev. B* **75** 184525