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Application of Gielis transformation to the design of metamaterial structures.

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Abstract. In this communication, the use of Gielis transformation to design more compact metamaterial unit cells is explored. For this purpose, transformed complementary split ring resonators and spiral resonators are coupled to micro-strip lines and theirbehaviour is investigated. The obtained results confirm that the useof the considered class of supershaped geometries enables the synthesis of very compact scalable microwave components.

1. Introduction

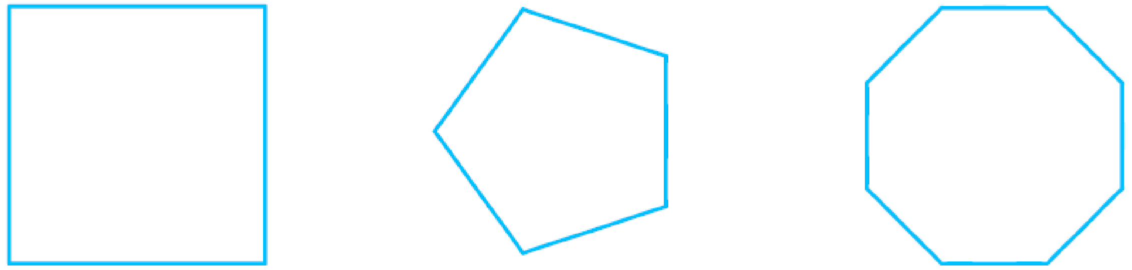
GielisTransformations (GT) [1] aregeometric transformations acting on planar functions f(θ) generalizing the Lamé equation that unifies a broad variety of natural and abstract shapes (Eq. 1). GTs can morph curves, such as circles, spirals or trigonometric functions, into infinite number of shapes, including regular polygons (Fig.1 a-c), starfish-like shapes (Fig. 1 d-f) and more complexspirals (Fig. 1 g-i).

$$k(\theta, a, b, m_1, m_2, n_1, n_2, n_3) = f(\theta)\left[\left|\frac{1}{a}\cos\left(\frac{m_1}{4}\theta\right)\right|^{n_2} + \left|\frac{1}{b}\sin\left(\frac{m_2}{4}\theta\right)\right|^{n_3}\right]^{n_1} \quad (1)$$

Since its introduction in the literature two decadesago, GTs have been extensively used in many fields of Science and Engineering for addressing a broad range of applications such as the solution ofboundary value problems [2, 3], pattern recognition [4], optimization of heat-shields in manned space vehicles [5], as well asin applied electromagnetics, for example in antennas [6, 7, 8, 9] and filterdesign [10].

The progressively more intensive use ofuse of GTs in engineering is mainly due to theireasy implementation in computer simulation tools, flexibility in design, as well asthe existence offamilies of shapes with specialproperties such as the class of closed curves featuring constant area but different lengths. These properties make GTs an ideal tool to study the effect of geometrical characteristics on the physical response of complex systems.

One meaningful examplein which the geometrical propertiesof the system play a main role in the definition of the relevant physical properties is related to metamaterials. Inthese type of structures a unit cell, in general resulting from the combination of different materials, e.g. conductors and dielectrics,is arranged periodically along a suitable lattice in such a way that new properties, such as negative-valued effective permittivity and/or permeability, emerge.This communication explores the application of GTs to two classes of metamaterial unit cells widely adopted in microwave filter design, namely complementary split ring resonators in Section 2 and spirals in Section 3.



a) $a;\ b = 1,\ m_1,m_2 = 4,\ n_1 = 5000,\ n_{2,};\ n_3=1150$ and $f[\theta] = 1, 0 \le \theta \le 2\pi$

b) $a;\ b = 1,\ m_1;m_2 = 5,\ n_1 = 5000,\ n_2;\ n_3 = 1150$ and $f[\theta] = 1, 0 \le \theta \le 2\pi$

c) $a;\ b = 1,\ m_1;m_2 = 8,\ n_1 = 5000,\ n_2;\ n_3 = 1150$ and $f[\theta] = 1, 0 \le \theta \le 2\pi$

d) $a;\ b = 1,\ m_1;m_2 = 4,\ n_1 = 10,\ n_2;\ n_3 = 30$ and $f[\theta] = 1, 0 \le \theta \le 2\pi$

e) $a;\ b = 1,\ m_1;m_2 = 5,\ n_1 = 10,\ n_2;\ n_3 = 30$ and $f[\theta] = 1, 0 \le \theta \le 2\pi$

f) $a;\ b = 1,\ m_1;m_2 = 8,\ n_1 = 10,\ n_2;\ n_3 = 30$ and $f[\theta] = 1, 0 \le \theta \le 2\pi$

g) $a;\ b = 1,\ m_1;m_2 = 4,\ n_1 = 10,\ n_2;\ n_3=2000$ and $f[\theta] = e^{-0.1\,\theta}, 0 \le \theta \le 6\pi$

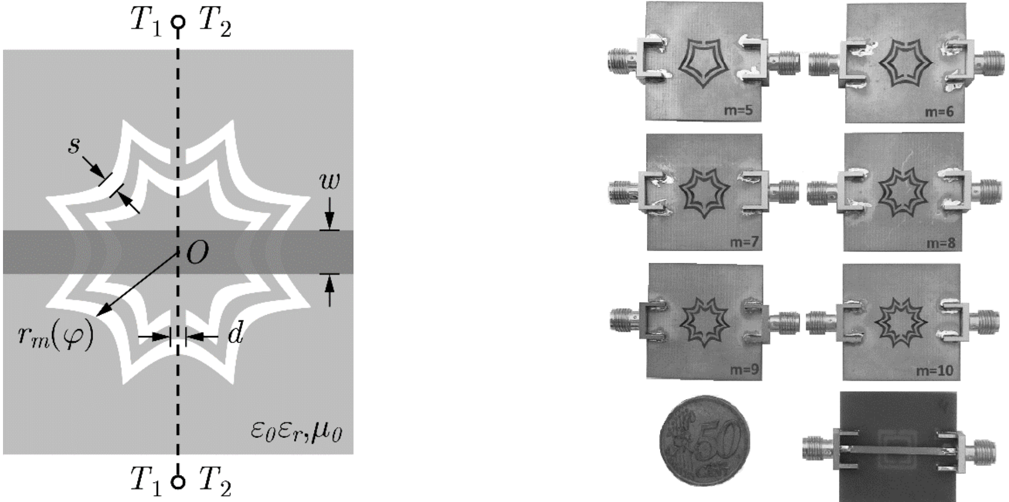
h) 1 $a;\ b = 1,\ m_1;m_2 = 5\ n_1 = 10,\ n_2;\ n_3 =2000$ and $f[\theta] = e^{-0.1\,\theta}, 0 \le \theta \le 6\pi$

i) $a=1;\ b = 2,\ m_1;m_2 = 8,\ n_1 = 600,\ n_2;\ n_3 =400$ and $f[\theta] = e^{-0.1\,\theta}, 0 \le \theta \le 6\pi$

Figure: 1 Transformations of a circle into regular polygons (a-c) and starfishes (d-f) and transformations of logarithmic spirals (g-i)

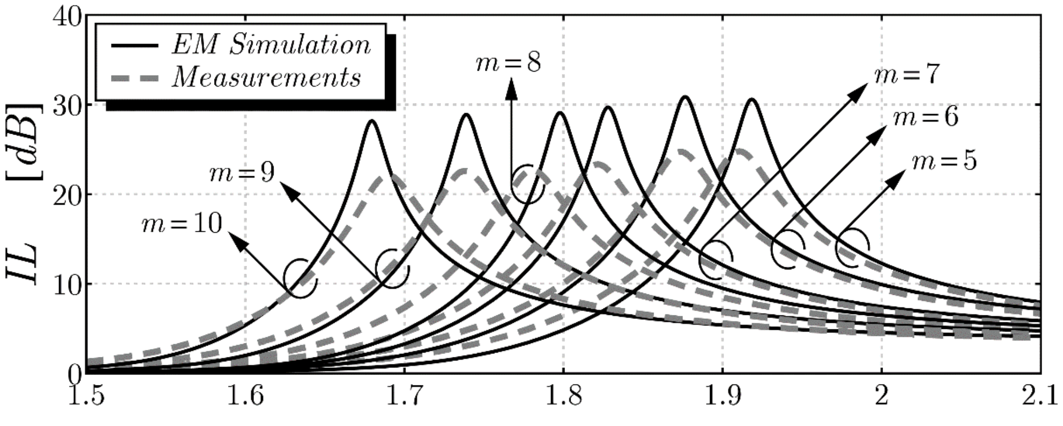
2. Gielis transformed Complementary split ring resonators

The use of split ring resonators (SRRs) and their complementary variant (CSRRs) as basic resonant units in planar microwave filters is being increasingly adopted due to their compact footprint compared to conventional resonators. This enables the realization of semi-lumped filtering structures with high performance and controllable characteristics [11, 12]. Typically, circular or square geometries are chosen for the design of the considered class of resonators, although other recent approaches use fractal or quasi-fractal geometries for the realization of miniaturized CSRRs, but still with a square-like form factor [13, 14].



a) Topology where s = d = 0.75 mm on a substrate with ε_r = 3.78 and tanδ = 0.025:

b) Prototypes with a; b = 1, n_1; n_2; $n_3 \to \infty$ and m = 5, 6, 7, 8, 9, 10.



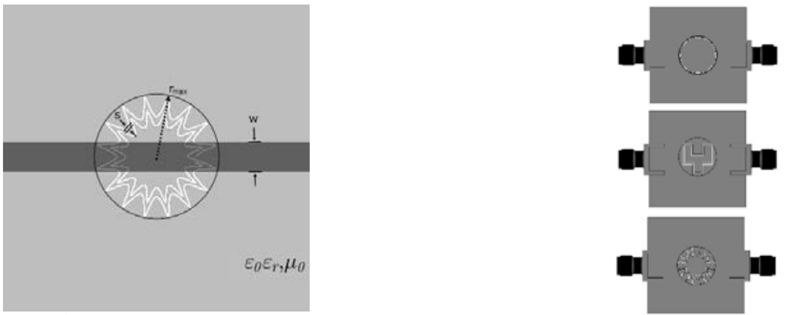
c) Comparison between simulated and measured insertion loss of the Gielis transformed CSSR single cell filter.

Figure: 2 Gielis transformed CSRR-based filters realized in microstrip technology.

Since resonance of CSRRs depends on the average length of the frequency slots and the area of the unit cell [15] one possible way to miniaturize CSRRS is to increase the length of the ring. A wellknown property of GT of circles is toincrease length by increasing the symmetry parameter *m* andkeep the rest of parameters unchanged.The enclosed area is then invariant. Making use of this property, the results on the band-stop properties of a microstrip line loaded with GT transformed CSRRs is shown in Figure 2a. Several prototypeswith increasing parameter *m* have been manufactured (Fig. 2b) and their insertion loss measured (Fig.2c). These results show that the electrical size of the unit cell gets larger with *m* as expected, based on the mathematical properties of GT.It is worth noting that, despite the size reduction, the fractional bandwidth is not significantly compromised.

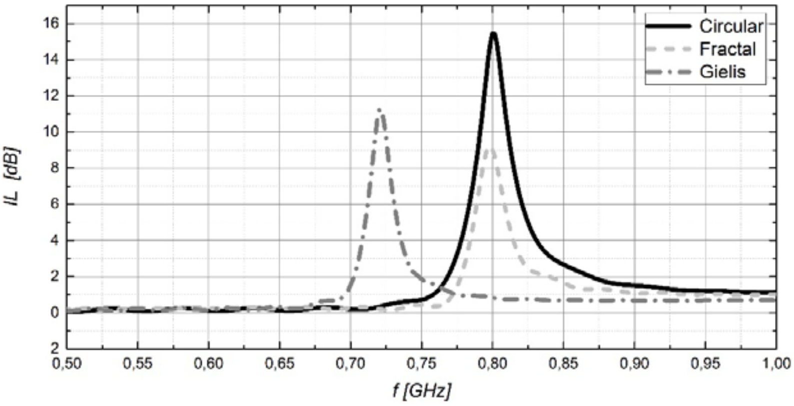
3. Gielis transformed logarithmic spirals.

Spiral resonators (SRs) are increasingly used as unit cells for metamaterials due to their furtherreduction of the electrical size. Following the same development trendsas for SRRs andCSRRs, the first SRs were based on canonical shapes [16], while later other more complexconceptsbased on fractals were also tested [17, 18]. To demonstrate the miniaturization capabilities ofGTs spirals, a comparison of a circular spiral, a fractal spiral as in [18] and a GT spiral hasbeen made with same maximum radius r_{max} (see Fig. 3a). The topology and the prototypes are shown in Figure 3a and 3b respectively, and simulation results are shown inFigure 3c.



a) Topology where s = d = 0,2 mm on a substrate with ε_r = 4.35 and tanδ = 0.035.

b) Spiral circular (top), Fractal [18] (center). and Gielis transformed spiral (bottom)



c) Simulated insertion loss of circular, fractal and GT spirals.

Figure: 3. Simulated insertion loss of circular, fractal and GT spiral.

The results displayed in Fig3.c show that transforming canonical spirals with GTs, electrically smaller unit cells can be designed without compromising bandwidth.

4. Discussion

GT have been used for metamaterials in the visible range [19], achieving double negative refractive index with supershaped unit cells, optimized via evolutionary algorithms on all parameters to operate in the blue region of the visible light spectrum. In this contribution, we show how the mathematical invariants of GT can be exploited to even reduce the electrical size of CSRRs and SRs unit cells. These structures show a reduction of around12% of the resonance frequency for the same maximum radius, and close to 30% when the same unit cell area is considered. These results are particularly relevant if the metamaterial has to be physically described as a continuous medium rather than as a discrete periodic structure.

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