Landau-level dispersion and the quantum Hall plateaus in bilayer graphene

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Abstract. We study the quantum Hall effect (QHE) in bilayer graphene using the Kubo-Greenwood formula. At zero temperature the Hall conductivity σ_{yx} is given by $\sigma_{yx} = 4(N+1)e^2/h$ with N the index of the highest occupied Landau level (LL). Including the dispersion of the LLs and their width, due to e.g. scattering by impurities, produces the plateau of the $n = 0$ LL in agreement with experimental results on doped samples and similar theoretical results on single-layer graphene plateaus widen with impurity concentration. Further, the evaluated resistivity ρ_{xx} exhibits a strong, oscillatory dependence on the electron concentration. Explicit results are obtained for δ-function impurities.

Keywords: bilayer graphene, quantum Hall effect, Hall resistivity
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INTRODUCTION

Charge carriers in single-layer graphene behave like *relativistic*, chiral massless particles with a light speed equal to the Fermi velocity ($\sim 10^6$ m/s) and possess a gapless spectrum that is linear in the wave vector near the K and K' points. In contrast, charge carriers in bilayer graphene are massive particles.

When a sufficiently clean single-layer graphene sheet is subjected to high magnetic fields, it exhibits a *half-integer* quantum Hall effect (QHE), $\sigma_{yx} = 4(N+1/2)e^2/\hbar$, that is very different from the conventional one $\sigma_{yx} = 2(N+1)e^2/\hbar$ in semiconductors, with N the index of the highest occupied LL. In contrast, in bilayer graphene we have $\sigma_{yx} = 4(N+1)e^2/\hbar$. So far several experiments have confirmed the appearance of the corresponding Hall plateaus at relatively high magnetic fields but also the existence of additional plateaux especially at zero gate voltage or electron concentration [1]. There can be various reasons for the n=0 LL plateau, e.g., electron-electron interaction [2]. In a previous paper [3] it was shown that the inclusion of the dispersion of the LLs and of their width, due to scattering by impurities, produces the plateau of the $n = 0$ LL in single-layer graphene because this scattering lifts the degeneracy of the spectrum with respect to the wave vector. Here we show that the same mechanism applies to bilayer graphene and produces the plateau of the corresponding $n = 0$ LL in agreement with experimental results [1]. We outline the formalism and present the results in Sec. II.

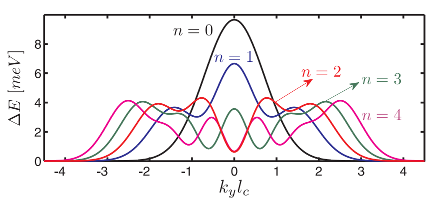

FIGURE 1. Energy dispersion of a few LLs vs $k_y \ell_c$ in bilayer graphene.

MODEL AND MAIN RESULTS

We describe charge carriers in bilayer graphene, in a perpendicular magnetic field B, using a 4×4 Hamiltonian. The eigenvalues, given in Ref. [4], are

$$\varepsilon_{n,s_1,s_2} = s_1 \left\{ t_c^2 + 2n + 1 + s_2 \sqrt{[t_c^4 + 2(2n+1)t_c^2 + 1]} \right\}^{1/2}; \tag{1}$$

here $n = 0, 1, 2, ..., s_1 = \pm 1, s_2 = \pm 1$, and $t_c = t/2^{1/2}$ with t being the interlayer coupling. For $s_1 = 1(-1)$ Eq. (1) gives the electron (hole) energy levels. Setting $t_c = 0$ in Eq. (1) gives the energy levels in single-layer graphene.

Within linear response theory the Hall conductivity σ_{yx} can be written in the form [3]

$$\sigma_{\mu\nu} = \frac{i\hbar e^2}{S_0} \sum_{\zeta \neq \zeta'} \frac{(f_\zeta - f_{\zeta'}) v_{\nu\zeta\zeta'} v_{\mu\zeta'\zeta}}{(E_\zeta - E_{\zeta'})(E_\zeta - E_{\zeta'} + i\Gamma_\zeta)}, \tag{2}$$

where S_0 is the area, $v_{\nu,\mu\zeta\zeta'}$ the matrix elements of the velocity operator, $\mu, \nu = x, y$, and $f_\zeta = f(E_\zeta)$ the Fermi-Dirac distribution function. Further, Γ is the LL width, $\beta = 1/k_B T$, and T is the temperature. To evaluate σ_{yx} we use Eq. (1) and the eigenfunctions of Ref. [4]. In the gauge $\mathbf{A} = (0, Bx, 0)$ the latter are given by $\Psi = [\varphi_a(x), i\varphi_b(x), \varphi_c(x), i\varphi_d(x)]^T e^{ik_y y}/\sqrt{L_y}$ with T denoting the transpose and $\varphi_i(x)$ the oscillator functions.

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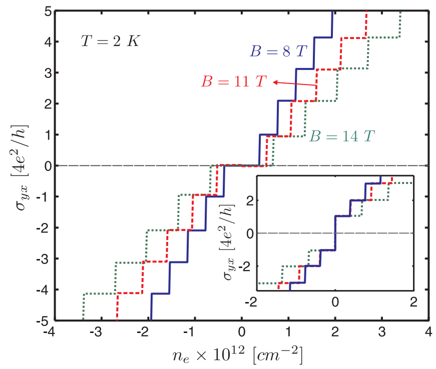

FIGURE 2. Hall conductivity σ_{yx} vs electron concentration n_e, with the energy correction included, for three magnetic fields $B = 8, 11$, and 14 T. Notice the plateau centered at $n_e = 0$. If this correction is neglected, the plateau at $n_e = 0$ disappears as shown in the inset.

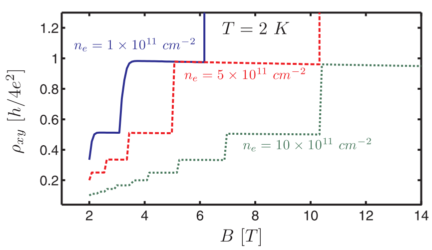

FIGURE 3. Hall resistivity ρ_{xy} vs magnetic field B for three electron concentrations n_e at $T = 2$ K.

Scattering by impurities also leads to a k_y-dependent shift or energy correction $\Delta\bar{E}_{n,s_1,s_2,k_y} = \langle n, s_1, s_2, k_y|U(\mathbf{r})|n, s_1, s_2, k_y\rangle$ of the eigenvalues given by Eq. (1), where $U(\mathbf{r})$ is the impurity potential, see also Ref. [3] for single-layer graphene. With $\xi = k_y\ell_c$, $\ell_c = \sqrt{\hbar/eB}$ the magnetic length, and $C = V_0/\ell_c L_y$ the result for δ-function potentials $U(\mathbf{r}) = V_0\delta(x)\delta(y)$ is (for $n > 0$)

$$\Delta\bar{E}_{n,s,k_y} = \frac{d_{n,s}^2 C e^{-\xi^2}}{2^n n!\sqrt{\pi}}\Big[(1+k_{n,s}^2)H_n^2(\xi) + (2n/\varepsilon_{n,s})^2 H_{n-1}^2(\xi) + (k_{n,s}/\varepsilon_{n,s})^2 H_{n+1}^2(\xi)\Big], \qquad (3)$$

where $C \approx 2\hbar v_F/l_c\sqrt{A}$, with A being a dimensionless parameter that depends on the density of impurities and their potential [5], $k_{n,s} = (\varepsilon_{n,s}^2 - 2n)/t\varepsilon_{n,s}$ and $d_{n,s} = \big[k_{n,s}^2[1+2(n+1)/\varepsilon_{n,s}^2] + 1 + 2n/\varepsilon_{n,s}^2\big]^{-1/2}$ with $s = \{s_1, s_2\}$. For $n = 0$ we obtain the simpler result $\Delta\bar{E}_{0,s,k_y} \approx Ce^{-\xi^2}/\sqrt{\pi}$ in which assuming $t^2 \gg 1$ lead to the same results for all $s_1 = \pm 1, s_2 = \pm 1$ cases. Equation (3) shows that the k_y degeneracy of the discrete LLs given by Eq. (1) is lifted, i.e., the LLs become narrow bands. We show that more clearly in Fig. 1 in which we plot a few LLs vs $k_y\ell_c$. A major consequence of Eq. (3) is the plateau centered at $n_e = 0$ and shown in Fig. 2 in which we plot the Hall conductivity σ_{yx} vs the electron concentration n_e for three magnetic fields $B = 8, 11$, and 14 T. Without the energy correction this plateau disappears as shown in the inset. The plateau behaviour in Fig. 2 also shows up when we plot σ_{yx} or the resistivity ρ_{yx} vs B for different electron concentrations. We show that in Fig. 3 for ρ_{yx} and three electron concentrations at $T = 2$ K.

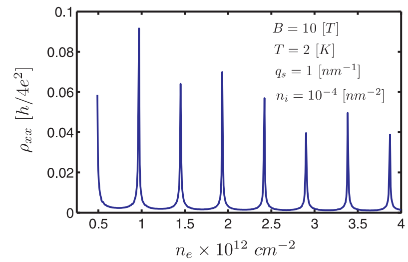

FIGURE 4. Longitudinal resistivity ρ_{xx} vs electron concentration for a field $B = 10$ T and $T = 2$ K.

In Fig. 4 we plot the longitudinal resistivity ρ_{xx} vs the electron concentration n_e for a magnetic field $B = 10$ T and $T = 2$ K. The other parameters are $A = 90$, screening wave vector $q_s = 1$ nm^{-1}, and impurity concentration $n_i = 10^{-4}$ nm^{-2}. The resistivity oscillates with the electron concentration as expected.

In summary we investigated the electron scattering by impurities on the Hall conductivity (σ_{yx}) in bilayer graphene, at low temperatures. This includes the correction $\Delta E_{n,s,k_y}$, given by Eq. (3), to the energy levels which becomes significant for $n = 0$ LL. This inclusion and the symmetry between electrons and holes give rise to the $n = 0$ Hall plateau. In addition, we showed that the longitudinal resistivity (ρ_{xx}) oscillates with electron concentration where the peaks values and the period of oscillations are linked to the scattering potential values.

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REFERENCES

1. E. V. Castro *et al.*, *Phys. Rev. Lett.* **99**, 216802 (2007).
2. Y. Barlas *et al.*, *Phys. Rev. Lett.* **101**, 097601 (2008).
3. P. M. Krstajić *et al.*, *Phys. Rev. B* **83**, 075427 (2011).
4. M. Zarenia *et al.*, *Phys. Rev. B* **85**, 075427 (2012).
5. Y. Zheng and T. Ando, *Phys. Rev. B* **65**, 245420 (2002).

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