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Fast generation of calculated ADF-EDX scattering cross-sections under channelling conditions

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Abstract

Advanced materials often consist of multiple elements which are arranged in a complicated structure. Quantitative scanning transmission electron microscopy is useful to determine the composition and thickness of nanostructures at the atomic scale. However, significant difficulties remain to quantify mixed columns by comparing the resulting atomic resolution images and spectroscopy data with multislice simulations where dynamic scattering needs to be taken into account. The combination of the computationally intensive nature of these simulations and the enormous amount of possible mixed column configurations for a given composition indeed severely hamper the quantification process. To overcome these challenges, we here report the development of an incoherent non-linear method for the fast prediction of ADF-EDX scattering cross-sections of mixed columns under channelling conditions. We first explain the origin of the ADF and EDX incoherence from scattering physics suggesting a linear dependence between those two signals in the case of a high-angle ADF detector. Taking EDX as a perfect incoherent reference mode, we quantitatively examine the ADF longitudinal incoherence under different microscope conditions using multislice simulations. Based on incoherent imaging, the atomic lensing model previously devel-

oped for ADF is now expanded to EDX, which yields ADF-EDX scattering cross-section predictions in good agreement with multislice simulations for mixed columns in a core-shell nanoparticle and a high entropy alloy. The fast and accurate prediction of ADF-EDX scattering cross-sections opens up new opportunities to explore the wide range of ordering possibilities of heterogeneous materials with multiple elements.

Keywords: Electron channelling, Scanning transmission electron microscopy (STEM), Annular dark field (ADF), Energy-dispersive X-ray spectroscopy (EDX), Scattering cross-section

1. Introduction

Despite their small size, nanostructured materials can display extraordinarily complex atomic structures associated with chemical inhomogeneities. Since their properties are fundamentally determined by the exact atomic arrangement, a quantitative structural characterisation in 3D is essential to get insight into the structural-properties relationship and hence the development of next-generation nanostructured materials. A popular characterisation technique is annular dark field scanning transmission electron microscopy (ADF-STEM) because of its sub-angstrom resolution in combination with its sensitivity to both the sample thickness and atomic number. To retrieve the 3D atomic structure, one can tilt the sample to different viewing directions and perform electron tomography. State-of-the-art ADF-STEM tomography has reached atomic resolution [1, 2]. In addition, from a single ADF-STEM image, it also has been demonstrated that one can determine the atomic column positions and count the number of atoms with high precision and accuracy for

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13 homogeneous materials [3, 4]. In combination with prior knowledge about the crystal periodic-
14 ity along the electron beam direction, atom counts can be translated into an initial atomic model,
15 which can be further optimised using an energy minimisation algorithm to obtain a low energy
16 state of the nanostructure [5]. A quantitative comparison study showed an excellent agreement be-
17 tween atomic resolution electron tomography and atom counting reconstructions [6]. This method
18 is dose-efficient since it only requires a single viewing direction. Therefore, it is suitable for the
19 3D characterisation of beam-sensitive materials and the investigation of particle dynamics at the
20 atomic scale during in-situ experiments [7, 8, 9].

21 To count the number of atoms from ADF-STEM images, we measure the so-called scattering
22 cross-section (SCS), corresponding to the total intensities of electrons scattered by a single atomic
23 column within the angular range of the ADF detector [10, 11]. This quantity outperforms peak
24 intensities because of its monotonic increase against the sample thickness and robustness against
25 various probe conditions (including defocus, source coherence, and aberrations) [11]. In prac-
26 tice, scattering cross-sections are measured by integrating the STEM signal over the Voronoi cell
27 for each atomic column [12] or by estimating the volume under a Gaussian peak that models an
28 atomic column shape [13]. If the experimental images are normalised against the incident beam,
29 the resulting scattering cross-sections can be quantitatively compared with simulated libraries ob-
30 tained under the same experimental conditions, enabling us to count the number of atoms in the
31 viewing direction for homogeneous materials. Alternatively, Van Aert et al. [14, 15, 16] proposed
32 a statistics-based method that decomposes the distribution of scattering cross-sections into over-
33 lapping normal distributions each corresponding to a specific number of atoms. One may further
34 combine the simulation and statistics-based method for a more reliable structural quantification

35 [15, 17]. For heterogeneous materials, the solution is often constrained in previous studies [10, 18]
36 by assuming a constant thickness and a linear dependence of the scattering cross-sections on the
37 chemical composition. However, this is only an approximation since the scattering cross-sections
38 depend on the location and the ordering of atoms in the column [19, 20, 21, 22, 23, 24]. Based on
39 the channelling theory of incoherent imaging, van den Bos et al. [25, 24] developed the so-called
40 atomic lensing model to take the ordering of multiple elements into account. This model predicts
41 the ADF scattering cross-section of a mixed column from the libraries of pure elements. When
42 including a priori knowledge about the sample, this was successfully applied to count the number
43 of atoms for an Au@Ag core-shell nanorod [24, 25]. To overcome the need for a priori knowledge
44 and to unscramble binary systems with mixed elements that are close in atomic number (Pt-Au for
45 example), it is difficult to rely on ADF-STEM images alone.

46 Energy dispersive X-ray (EDX) spectroscopy and electron energy loss spectroscopy (EELS)
47 can fingerprint different elements. With modern instrumentation, the acquisition of EDX and
48 EELS spectrum imaging datasets at atomic resolution is now becoming more routinely possible.
49 The synchronisation of the signals between the probe scanning system and different detectors al-
50 lows simultaneous acquisition of ADF-EDX-EELS hence maximising the transfer of structural and
51 chemical information [26, 27]. In addition, fast-scan multi-frame imaging techniques can mitigate
52 scan noise (both linear and non-linear), reduce the sample damage, and improve the signal-to-noise
53 ratio [28, 29]. The fast-evolving detector design also leads to an ever-changing detector geometry
54 and efficiency [30], which needs to be accounted for quantitatively when calibrating EDX signals
55 to the absolute scale [31, 32, 33]. To overcome the difficulties in the characterisation of the EDX
56 detectors, we can incorporate the experimentally measured EDX partial cross-section, which is

57 called a *partial* scattering cross-section since it includes the microscope-dependent factors during
58 normalisation [34].

59 Even though atomic resolution spectroscopy has gradually improved from the experimental
60 side and inelastic scattering calculations within the multislice framework are well-established (see
61 review [35] and references therein), difficulties for quantification persist. If we want to quantify
62 spectroscopy data alongside ADF using similar quantification routines, we need to include the
63 effects of channelling in the spectroscopy simulations. The channelling effect originates from
64 the fact that a fast negatively charged electron will be attracted by the positively charged atomic
65 nuclei. As a consequence, an atomic column with periodically spaced atoms along the beam
66 direction acts as a waveguide dynamically focusing the electrons. This leads to a non-linear sig-
67 nal as a function of depth for atomic resolution ADF and EDX, which significantly complicates
68 composition quantification. Although both high-angle ADF and EDX are known to be highly lo-
69 calised and incoherent, it is unclear whether they follow the same channelling behaviour. Since the
70 EDX signal is fully incoherent, the EDX-ADF comparison allows an investigation of the degree
71 of ADF longitudinal incoherence [36]. In addition, the number of possible configurations grows
72 exponentially with the number of different types of elements and thickness of the sample, hence
73 quickly exceeding the computation time of multislice calculations. Therefore, MacArthur et al.
74 [23, 37] suggested tilting the sample by $2-3^\circ$ to reduce the effect of channelling to perform EDX
75 quantification, which is at the cost of resolution. To have both the atomic resolution and compu-
76 tational feasibility in the presence of channelling, the applicability of the atomic lensing model
77 to efficiently predict EDX scattering cross-sections of mixed columns will be investigated. This
78 model has previously been developed to predict ADF scattering cross-sections of mixed columns

79 [24, 25]. Since its origin is based on longitudinally incoherent imaging, it is expected that this
80 method will be applicable for fast EDX predictions.

81 Due to electron channelling complicating elemental quantification, special attention to the in-
82 coherence of ADF and EDX image formation is needed. In optics, coherence is caused by the
83 interference of wavefunctions upon signal generation. Conventionally, the so-called incoherent
84 imaging mode of ADF [38] refers to transverse incoherence expressing that the image intensity
85 can be written as a convolution of the probe intensity and the object function being peaked at the
86 atomic column positions. Transverse incoherence not only yields a directly interpretable image
87 but also allows us to associate the scattered intensities with atomic columns, enabling the quan-
88 tification of scattering cross-sections [11]. Less visited is the longitudinal incoherence expressing
89 that the image intensity can be written as an incoherent summation of signals generated along
90 depth as defined in [39]. The EDX signal is known to be fully incoherent, both transversely and
91 longitudinally, as summing over all possible final states and integrating over the full energy loss
92 and momentum space [40, 39]. The story can be different when integrating over part of the mo-
93 mentum space with a finite energy window as in EELS. Dwyer [41] examined the longitudinal
94 coherence of EELS with varying collection angles, which enables the decoupling of the inelastic
95 signal from elastic scattering after the ionisation event in later experimental studies [42, 43]. The
96 ADF signal, similarly, only collects electrons scattered within the detector, the coherence of which
97 needs further examination. Since ADF intensities are dominated by thermally scattered electrons
98 associated with random phase shifts of transmission functions, one may well suspect that the ADF
99 signal is transverse incoherent due to phonon scattering [44]. Later analysis [45, 46, 47] showed
100 that phonon scattering is not a prerequisite for transverse incoherent imaging. In fact, transverse

101 incoherence is established due to the geometry of the ADF detector. The integration over the de-
102 tector removes the sensitivity to coherent interference effects [48]. However, the detector itself is
103 not efficient in destroying the coherence along the electron beam direction – which we refer to as
104 longitudinal incoherence – where phonon scattering will have a more significant effect.

105 The present paper aims to address the following key questions related to ADF-EDX quan-
106 tification under channelling conditions: (a) Do EDX and ADF scattering cross sections have the
107 same thickness scaling behaviour due to channelling? (b) How does the longitudinal incoherence
108 of ADF compare to EDX as a function of ADF collection angles? (c) How can the atomic lens-
109 ing model be used to predict EDX scattering cross-sections for mixed columns? In section 2,
110 we will discuss the origin of the incoherence for ADF and EDX signals in the multislice frame-
111 work. In section 3, we will examine the longitudinal incoherence of ADF signals by simulating
112 the ADF-EDX scattering cross-sections under different microscope conditions. In section 4, we
113 will expand the atomic lensing model to spectroscopy enabling a fast prediction of EDX scattering
114 cross-sections of mixed columns.

115 **2. Electron scattering theory for ADF and EDX within the multislice framework**

116 By dividing materials into slices, the multislice algorithm describes multiple scattering as a
117 repetition of transmission within each slice and free propagation between slices. In this section,
118 we will briefly outline the equations for ADF and EDX signals to understand their relationship,
119 while readers are referred to Kirkland’s book on the full topics of multislice [49] and the review
120 by Dwyer on the inelastic scattering [35].

121 The relativistically-corrected Schrödinger equation for a fast electron traveling in the forward

122 direction z [50] can be written as:

$$\frac{\partial \psi(\mathbf{r}, \mathbf{R}, z)}{\partial z} = \left[\frac{i\lambda}{4\pi} (\nabla_{\mathbf{r}}^2) + i\sigma V(\mathbf{r}, z) \right] \psi(\mathbf{r}, \mathbf{R}, z), \quad (1)$$

123 where $\psi(\mathbf{r}, \mathbf{R}, z)$ is the electron wave at thickness z , probe position \mathbf{R} and real space 2-D coordinate
124 vector $\mathbf{r} = (x, y)$. The impact parameter is $\sigma = me\lambda/2\pi\hbar^2$, $V(\mathbf{r}, z)$ is the electrostatic potential at
125 depth z , e is the electron charge, m and λ are the relativistically corrected electron mass and
126 wavelength, respectively. Once the electron wave reaches the exit surface, it propagates to the
127 detector plane in the far field. The intensity scattered within the inner and outer collection angle
128 of the ADF detector will be collected:

$$I_{ADF}(\mathbf{R}) = \int D(\mathbf{k}) |\psi(\mathbf{k}, \mathbf{R}, z)|^2 d\mathbf{k}, \quad (2)$$

129 where $\psi(\mathbf{k}, \mathbf{R}, z)$ is the Fourier transform of $\psi(\mathbf{r}, \mathbf{R}, z)$, $D(\mathbf{k})$ is the ADF detector response which
130 can be characterised experimentally as an input for simulation. In this study, we assume an ideal
131 detector sensitivity with $D(\mathbf{k})$ equal to 1 for points \mathbf{k} on the detector and 0 otherwise in the diffrac-
132 tion space.

133 Since the incident electrons travel fast as compared to the vibration period of the atoms, the
134 atoms are seen as a frozen snapshot. Therefore, in the frozen phonon approach, the observed
135 electron intensity distribution $|\psi(\mathbf{k}, \mathbf{R}, z)|^2$ in Eq. 2 is calculated for many different atom config-
136 urations following the Einstein model and the resulting intensity distributions are averaged over
137 time. Although the Einstein model cannot describe the vibrational modes in low-loss EELS spec-
138 trum [51] (which needs a correlated vibrational model), the predicted integrated ADF intensity due
139 to phonon excitation is correct. Frozen phonon calculations allow us to separate the elastic and

140 thermally scattered electrons. Following Ref. [52], the exit wavefunction in reciprocal/real space
 141 can be expressed as:

$$\psi(\mathbf{k}/\mathbf{r}, \tau) = \langle \psi(\mathbf{k}/\mathbf{r}, \tau) \rangle + \delta\psi(\mathbf{k}/\mathbf{r}, \tau), \quad (3)$$

142 where \mathbf{k}/\mathbf{r} is either the reciprocal/real space vector as defined previously, τ represents a frozen
 143 phonon configuration of atom positions, $\langle \rangle$ is the average operation over different phonon config-
 144 urations and $\delta\psi(\mathbf{k}/\mathbf{r}, \tau)$ is the deviation from the average wavefunction for a particular phonon
 145 configuration. The total intensity $\langle |\psi(\mathbf{k}/\mathbf{r}, \tau)|^2 \rangle$ is the incoherent sum of electrons averaged over
 146 the phonon configurations:

$$\underbrace{\langle |\psi(\mathbf{k}/\mathbf{r}, \tau)|^2 \rangle}_{\text{Total}} = \underbrace{|\langle \psi(\mathbf{k}/\mathbf{r}, \tau) \rangle|^2}_{\text{Elastic}} + \underbrace{\langle |\delta\psi(\mathbf{k}/\mathbf{r}, \tau)|^2 \rangle}_{\text{TDS}}. \quad (4)$$

147 In this equation, the elastic scattering contribution $|\langle \psi(\mathbf{k}/\mathbf{r}, \tau) \rangle|^2$ is the modulus square of the
 148 averaged wavefunction and the thermal diffuse scattering (TDS) contribution $\langle |\delta\psi(\mathbf{k}/\mathbf{r}, \tau)|^2 \rangle$ is
 149 the average of the modulus square of the wavefunction deviations. When substituting Eq. 4 in
 150 Eq. 2, the elastic and TDS contributions to the ADF signal can be separated.

151 For a quantum mechanical view of treating phonons, the electron intensity can be considered
 152 as the incoherent sum of electrons scattered from different initial states of phonons according to
 153 their probability distribution, known as quantum excitation of phonons (QEP) [53]. The observed
 154 electron intensities I_{total} is calculated as the incoherent sum weighted over the initial phonon dis-
 155 tribution [53]:

$$I_{total}(\mathbf{k}/\mathbf{r}) = \int |\psi(\mathbf{k}/\mathbf{r}, \tau)|^2 P(\tau) d\tau. \quad (5)$$

156 Under the Einstein phonon model, the probability distribution $P(\tau)$ is defined as:

$$P(\tau) = \frac{1}{\sqrt{2\pi\langle u^2 \rangle}} \exp\left[-\frac{(\tau - \tau_0)^2}{\langle u^2 \rangle}\right], \quad (6)$$

157 where τ and τ_0 are the current and equilibrium atom position respectively, and $\langle u^2 \rangle$ is the mean
158 squared displacement of the atom. The elastic contribution $I_{elastic}$ is the modulus square of the
159 average of the wavefunctions over the phonon distributions [53]:

$$I_{elastic}(\mathbf{k}/\mathbf{r}) = \left| \int \psi(\mathbf{k}/\mathbf{r}, \tau) P(\tau) d\tau \right|^2. \quad (7)$$

160 The TDS contribution is simply the difference between the total intensity I_{total} and the elastic
161 contributions $I_{elastic}$. From Eq. 3-7, it follows that the QEP approach is numerically equivalent
162 to the frozen phonon approach but with different underpinning concepts [52, 53]. Specifically,
163 for a single electron, QEP considers all phonon configurations through the distribution function
164 $P(\tau)$. In contrast, the frozen phonon approach treats a single electron scattered from only one
165 phonon configuration. Nevertheless, the QEP/frozen phonon approaches both calculate the TDS
166 by explicitly subtracting the coherent contributions from the total intensities. Thus, TDS can be
167 considered incoherent in all respect. The linearity of comparing ADF and EDX cross-sections
168 depends on the collection angles where TDS dominates the ADF intensities.

169 The ADF intensities can also be calculated with the absorptive potential approach [54, 55].
170 In this approach, the ADF longitudinal incoherence is embedded with thermal diffuse scattering
171 using the same equation as EDX but with an effective TDS potential in Eq. 9. Therefore, we
172 can already predict a linear correlation between ADF and EDX and hence between their cross-
173 sections in the presence of channelling. However, two differences are observed: (a) the effective
174 potential is different for ionisation and TDS which depends on the ADF detector geometry and

175 (b) the phonon scattered electrons can still excite X-rays. An inherent drawback of the absorptive
 176 potential approach is that once electrons are absorbed, further elastic or inelastic scattering of the
 177 thermally scattered electrons is not accounted for in the simulation and consequently does not
 178 properly describe the multiple scattering in a thick sample [56]. A Detailed comparison study
 179 between the incoherent absorptive potential and frozen phonon can be found in [57]. Therefore,
 180 we will take the frozen phonon and numerically equivalent QEP approach in this study.

181 A fast electron can also excite atomic inner-shell electrons to higher unoccupied states fol-
 182 lowed by de-excitations via Auger electrons or characteristic X-ray emissions. The EDX effective
 183 potential calculates the transition probabilities with all possible energy-momentum transfers and
 184 all final continuum states explicitly summed up [40, 35, 56]:

$$V_{EDX}(\mathbf{r}, z) = \frac{\pi m}{h^2} \sum_n \frac{1}{k_n} |H_{n0}(\mathbf{r}, z)|^2, \quad (8)$$

185 where H_{n0} is the projected transition matrix element of a core-shell electron excited from the initial
 186 state $|0\rangle$ to final state $|n\rangle$ with certain energy loss, $k_n = \frac{1}{\lambda_n}$ is the wave number of the inelastically
 187 scattered electron associated with the $|0\rangle$ to $|n\rangle$ excitation. The EDX signal can be considered as
 188 the cumulative sum of the probe convoluted with the effective potential at each thickness, resulting
 189 in an incoherent form for image formation:

$$I_{EDX}(\mathbf{R}) = \frac{4\pi}{h\nu} \sum_z \int V_{EDX}(\mathbf{r}, z) |\psi(\mathbf{r}, \mathbf{R}, z)|^2 d\mathbf{r}. \quad (9)$$

190 $V_{EDX}(\mathbf{r}, z)$ is the EDX effective ionisation potential projected for a single plane of atoms at a depth
 191 z for a particular X-ray emission. Note that EDX is influenced by dynamical scattering before
 192 ionisation with the altered probe intensity convolves with the EDX effective potential. The elastic
 193 scattering after ionisation has no further consequences in EDX, which is different from the double

194 channelling situation for EELS. Therefore, the EDX intensities can be written as a summation of
195 the sample thickness for each element at each slice and are longitudinally incoherent. Here, we
196 assume that all excited states for the targeted orbital at the ground state lead to the generation
197 of an X-ray and that the detector reaches the full solid angle. In practice, for full quantification
198 of EDX signals, we should also consider (a) the fluorescence yield of X-rays, (b) the detector
199 geometry, efficiency, and shadowing [31] and (c) the absorption and scattering of X-rays in their
200 pathway toward the detector [33]. To simplify the quantification, the effects (a) and (b) simply
201 scale Eq. 9 and can be taken into account using the microscope-dependent partial cross-section
202 [34]. Absorption (effect (c)) is usually negligible for nanostructured materials due to its small size
203 but should be considered when its effect cannot be ignored in some systems (Ni-Al for example)
204 due to the strong absorption among different elements. One can check the database in [58] if
205 strong X-ray interaction exists in the system of interest.

206 We should note that the coupling between ADF and EDX may be more subtle than convention-
207 ally assumed [59]. A small proportion of phonon scattered electrons is also involved in ionisation
208 events. Those electrons lose a significant amount of energy and momentum, thus changing the
209 observed electron density distribution in momentum space for ADF and EELS detectors. The
210 implication of ionisation in HAADF and phonon spectroscopy is discussed in [59], showing a
211 difference of scattering cross-sections between QEP multislice calculations with and without in-
212 cluding the contribution of ionisation. Given the small ionisation cross-sections and the size of
213 nanostructures, this effect is not included in this study.

214 In this study, we used muSTEM [56] to simulate the CBED, ADF, and EDX signals for pure
215 elements in Section 3 to compare their channelling behaviours. To ameliorate the memory re-

216 quirement, muSTEM augments phonon configurations by random translation of pre-calculated
217 transmission functions by an integer number of unit cells in each direction, which makes it not
218 suitable for non-periodic structures. Since the on-the-fly calculation is not accessible in the cur-
219 rent version of muSTEM, a large amount of pre-calculated transmission functions without random
220 phase translation is still doable for small nanoparticles as performed before [23] but not feasible
221 for thick high entropy alloys in this study. Therefore, we take the EDX effective potential based on
222 the inelastic scattering factor tabulated in muSTEM [56, 40] and then implemented it in MULTEM
223 [60, 61] for benchmark in Section 4.2 and for the high entropy bulk alloys in Section 4.3. Note
224 that our EDX implementation is still at the proof-of-concept stage and not yet optimised for GPU
225 acceleration. Thus, for small core-shell nanoparticle case studies, we still used muSTEM.

226 **3. Relationship between ADF-EDX scattering cross-sections**

227 ADF and EDX have a non-linear relationship against thickness [25] due to dynamical electron
228 scattering, particularly at the atomic scale in zone-axis orientation. This is clear from Fig. 1(a),
229 where the ADF and EDX scattering cross-sections are calculated using multislice for a pure Au
230 crystal and normalised against the corresponding values of a single atom. Although the ADF
231 and EDX demonstrate channelling behaviour with both non-linear scattering cross-section curves
232 against sample thickness, they simply differ by a scaling factor. Here we employed a 300 keV
233 aberration-corrected probe with a convergence semi-angle of 20 mrad and ADF collection semi-
234 angle of 50-150 mrad. The detailed settings can be found in Table 1 and will be used for the
235 following simulations in this study if not stated otherwise. We also included the thermal vibration
236 root-mean-squared displacement and X-ray line information for each element in Table 2. As shown

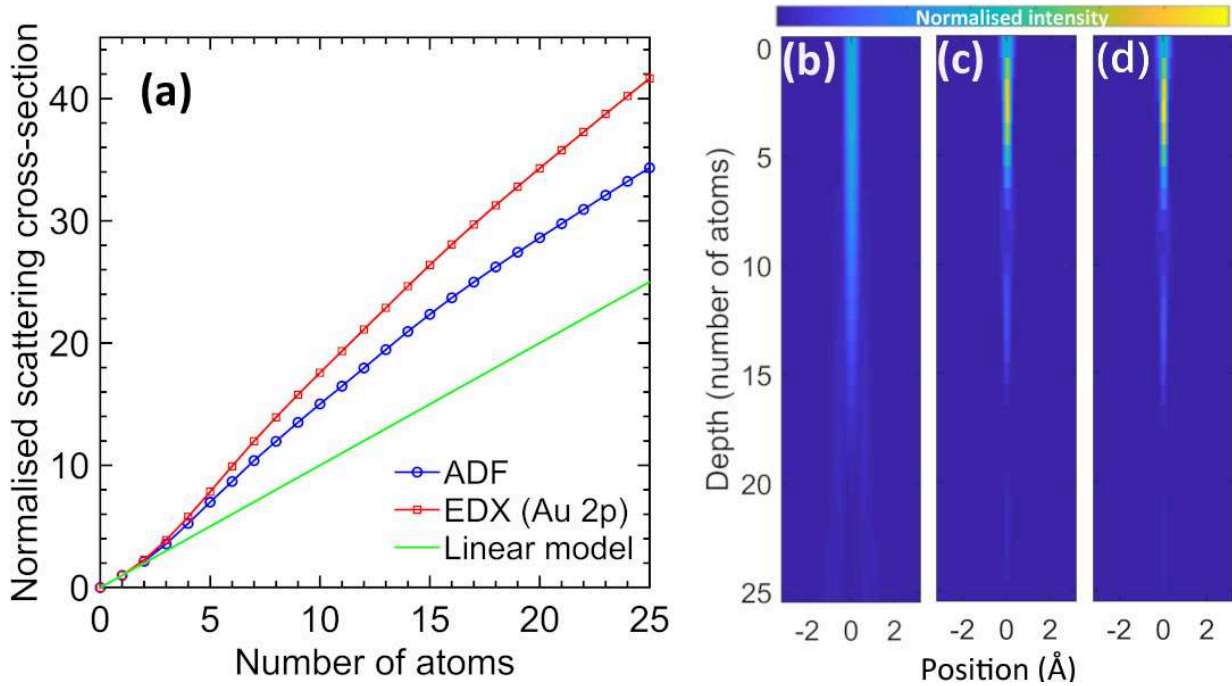


Figure 1: (a) Plots of ADF (with collection semi-angle of 50-150 mrad) and EDX (using transition potential of the 2p orbital, corresponding to the Au L peak) scattering cross-sections as a function of the number of atoms for an Au face-centred cubic (FCC) crystal in [1 0 0] direction. The scattering cross-sections are normalised against those of single atoms and compared with the linear model. Cross-sectional depth profile of the electron probability for an aberration-corrected probe in (b) vacuum, (c) a single isolated Au atomic column, and (d) an Au atomic column in a crystal.

237 in Fig. 1(a), ADF and EDX scattering cross-sections have a clear deviation from the linear model
238 even for a very thin sample. This can be understood by examining the depth profile of the electron
239 probe free propagation in the vacuum and comparing it to that along a single isolated atomic
240 column and an atomic column in a crystal, Fig. 1(b-d). The presence of atoms focuses the electron
241 probe – for instance, the probe is narrower with a higher electron density especially for the first few
242 atoms in (c-d) compared to in vacuum (a) in Fig. 1 – since their positive nuclei act as atomic lenses
243 for the negatively charged electrons, known as electron channelling. A strongly focused probe
244 leads to higher yields of EDX and ADF scattering cross-sections, which vary along the electron
245 beam direction due to dynamic scattering. For a well-separated lattice or more importantly a thin
246 sample, the coupling between columns is not significant so the electron channelling is largely
247 confined to a single column [62]. This behaviour is therefore similar for the isolated column
248 and the full lattice, as shown in Fig. 1(c-d). The picture for closely-spaced atomic columns in a
249 thick sample is different since the electron beam may channel, for instance, between the dumbbell
250 structure in Si at larger depths [63].

251 Although Fig. 1(a) shows that ADF and EDX have a non-linear relationship against sample
252 thickness, we might expect the two signals to follow an identical trend if they are fully incoherent.
253 To test the ADF longitudinal incoherence as a function of scattering angles, we examined the
254 dependence between the two signal modes numerically using multislice calculations. Position
255 averaged convergent beam electron diffraction (PACBED) patterns were computed together with
256 EDX for a unit cell in a pure Au crystal with thicknesses of 1-25 atoms (corresponding to 0-10 nm).
257 By radially integrating a PACBED pattern in the azimuthal direction and dividing by the number of
258 atomic columns in the scanned area, angular resolved scattering cross-sections are obtained, which

Table 1: Settings used for multislice simulations.

Acceleration voltage	300 kV
Defocus	0 nm
Spherical aberration	0 mm
Convergence semi-angle	20.0 mrad
Potential pixel size	4.38 pm
STEM image pixel size	0.24 Å
ADF detector angle	50 – 150 mrad
Number of phonon configurations	30

Table 2: Thermal vibrations, EDX lines and cross-sections for different elements.

Element	Root-mean-squared displacement (Å)	X-ray line	X-ray energy (keV)	Orbital excited	ionisation energy (keV)	ionisation section (Å ²)	cross-
Al	0.1012	K_{α}	1.486	1s	1.560	1.67×10^{-5}	
Ag	0.0966	L_{α}	2.984	2p	3.524	2.12×10^{-5}	
Pt	0.0686	L_{α}	9.441	2p	11.564	4.47×10^{-6}	
Au	0.0884	L_{α}	9.712	2p	11.919	4.30×10^{-6}	

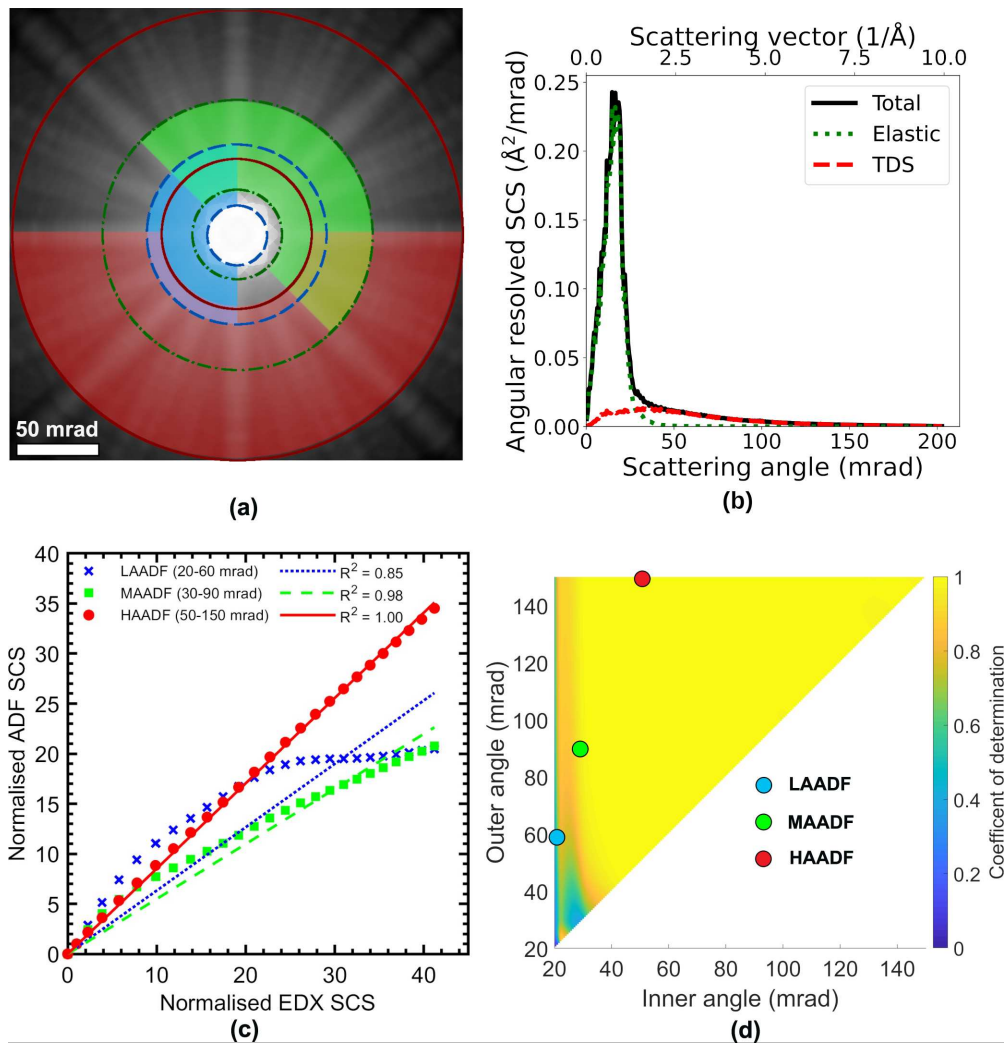


Figure 2: (a) PACBED pattern (shown on a log scale) to demonstrate the range of the LAADF (20-60 mrad), MAADF (30-90 mrad), HAADF (50-150 mrad) detectors. As those detectors overlap, only half of the detectors are colored for better visualisation of their collection angles with the other half indicated by solid or dashed lines. (b) Angular resolved scattering cross-section (including total, elastic, and TDS contributions) as a function of scattering angle (in mrad) or scattering vector (in $1/\text{\AA}$). (c) LAADF, MAADF and HAADF scattering cross-sections as a function of the normalised EDX scattering cross-sections together with a linear regression line. (d) Coefficient of determination R^2 of the ADF-EDX linear dependence for a range of different inner and outer collection angles. The simulations were performed for an Au crystal in a $[0\ 0\ 1]$ direction with varying thicknesses (1-25 atoms), illuminated using 300 keV electrons with a 20 mrad condenser aperture and no lens aberrations.

259 are then integrated for all possible inner and outer collection angles to obtain the corresponding
 260 ADF scattering cross-sections. For instance, three typical ranges for low angle (LAADF 20-60
 261 mrad), medium angle (MAADF 30-90 mrad), and high angle ADF (HAADF 50-150 mrad) are
 262 shown in Fig. 2(a). The contribution of elastic scattering and TDS to the total cross-sections are
 263 separated according to Eq. 5-7 in Fig. 2(b), where we can see that the TDS dominates from 50
 264 mrad or 2.5 \AA^{-1} . This operation is applied to all PACBED patterns at different thicknesses and the
 265 retrieved ADF scattering cross-sections are plotted against EDX scattering cross-sections for the
 266 same column thickness in Fig. 2(c). These ADF and EDX scattering cross-sections are fitted using
 267 linear regression. Whereas HAADF has a perfect linear dependence against EDX for different
 268 thicknesses, LAADF and MAADF do not show such a relationship. It is worth mentioning that
 269 the red curve (HAADF 50-150 mrad) contains the same ADF and EDX values as in Fig. 1(a). The
 270 goodness of fit of the linear regression model can be quantitatively measured by the coefficient of
 271 determination R^2 , which is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^n (\sigma_i - \sigma_i^{lin})^2}{\sum_{i=1}^n (\sigma_i - \bar{\sigma})^2}, \quad (10)$$

272 with σ_i the simulated ADF cross-section, σ_i^{lin} the predicted ADF value based on linear regression,
 273 and $\bar{\sigma}$ the mean value of the simulated ADF cross-sections. A perfect linear dependence between
 274 the ADF and EDX signals means that the R^2 value equals 1. Fig. 2(d) shows the R^2 value as a
 275 function of the inner and outer detector angle. Since the EDX signal is perfectly incoherent, this
 276 graph may be considered an ADF longitudinal incoherence map. The results reassure our common
 277 understanding that the HAADF signal is incoherent while signals recorded at low angles are not.
 278 Note that the ADF coherence measured in this approach depends on the sample and microscope

279 parameters. For instance, an ADF detector being incoherent for a thin sample with light elements
 280 may become semi-coherent for a thick sample with heavy elements.

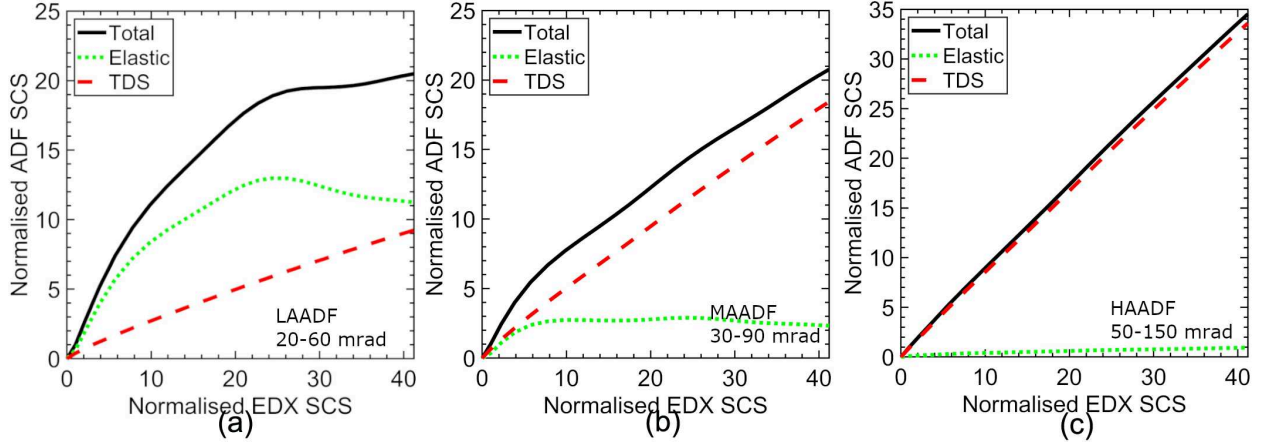


Figure 3: Plots of normalised ADF scattering cross-sections against EDX scattering cross-sections for (a) LAADF, (b) MAADF, and (c) HAADF.

281 To understand the deviation of the ADF signal from perfect incoherence at low and medium
 282 angles, we can separate the contributions of elastic scattering and thermal diffuse scattering in the
 283 diffraction patterns according to Eq. 4. As shown in Fig. 3(a-b), the elastic signal has a significant
 284 contribution at low and medium angles of the ADF detector resulting in a deviation of the linearity
 285 against EDX. In contrast to the elastic contribution, phonon scattered signals are almost linear
 286 against EDX with increasing thickness and dominate the HAADF intensities as shown in, Fig. 3(c).

287 To investigate the longitudinal incoherence with varying voltage, the ADF collection range is
 288 measured in terms of the scattering vector in \AA^{-1} and the geometric angle in mrad. As shown
 289 in Fig. 4, the ADF-EDX linear dependence of conventionally considered HAADF angle (50-150
 290 mrad) at 300 kV could break down at 60 kV. In contrast, the linearity can be well-kept when we
 291 translate the collection angle of 50-150 mrad to $2.53\text{-}7.62 \text{\AA}^{-1}$ at 300 kV and apply it for a lower

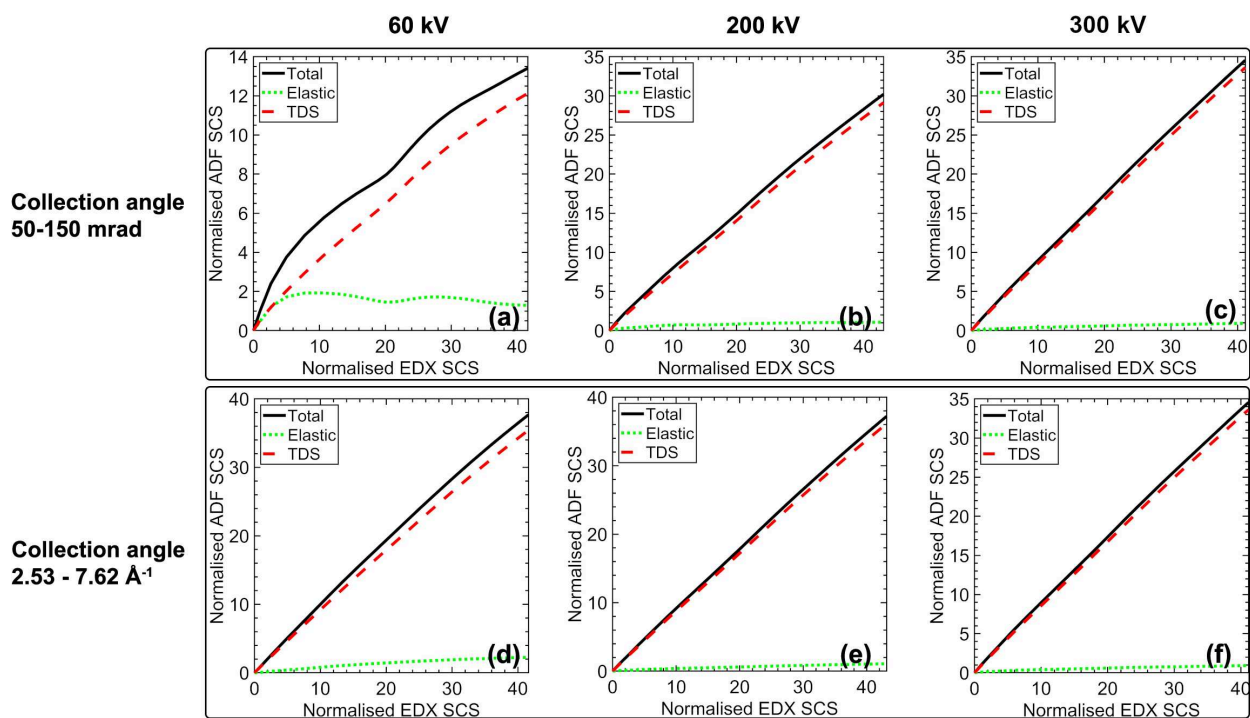


Figure 4: Plots of normalised ADF scattering cross-sections against EDX scattering cross-sections for different acceleration voltages with (a-c) the same collection angle in mrad; (d-f) the same collection angle in $1/\text{\AA}$.

292 voltage. The reason is that the positions of CBED disks for elastic scattering are controlled by
293 the lattice spacing while the phonon scattering is characterised by root-mean-square displacement
294 of the element, both are constants measured by the \AA^{-1} in the diffraction plane. Further angular
295 resolved scattering cross-section calculations show that the range where the thermal diffuse scat-
296 tering starts to dominate is relatively invariant to the acceleration voltage. Though for the case of
297 60 kV (2.53-7.62 \AA^{-1} or equivalently 123-370 mrad), ADF scattering cross-sections have a small
298 but not negligible contribution from elastic signals, its relationship against EDX is still linear. The
299 elastic contribution, in this case, is due to the first-order Laue zone, which falls within the ADF
300 range at low voltage.

301 In this section, we showed that the integration over the ADF detector, which destroys the
302 transverse coherence, does not control the longitudinal coherence. One must select a sufficiently
303 high inner collection angle to make the truly incoherent phonon scattered electrons dominate the
304 ADF signal. Note that in this simulation study, we followed the conventional uncorrelated Einstein
305 model of phonons generation that displaces atoms in 2D. The root-mean-square displacement
306 $\sqrt{\langle u^2 \rangle}$ for different elements used in this study are given in Table 2. The proper 3D phonon with
307 realistic dispersion, which includes a greater excitation of long-wavelength correlated phonons, is
308 beyond the scope of this study.

309 **4. Extending the atomic lensing model for spectroscopy**

310 In the previous section, we examined a robust linear dependence between EDX and ADF
311 via multislice simulations. However, we should note that such a simulation is computationally
312 expensive. For a 20-atom-thick binary alloy, there are more than 1 million different 3D column

313 configurations to cover the entire composition range. The situation is even worse when the number
314 of elements further increases. Therefore, to quantify EDX at atomic resolution, a fast prediction
315 method is needed for the elemental quantification taking dynamical diffraction into account. The
316 atomic lensing model, which is a non-linear model under channelling conditions, was previously
317 developed for ADF and successfully applied in atom counting of mixed columns in an Au@Ag
318 core-shell nanoparticle [24, 25]. Based on the incoherent imaging of ADF and EDX signals, one
319 would expect that this model also works for EDX. In section 4.1, the theoretical extension of
320 the atomic lensing model to EDX is described. Section 4.2 will benchmark the computational
321 complexity, speed, and accuracy of the atomic lensing model compared to the multislice and the
322 recently developed PRISM algorithm [64]. Then, in Section 4.3, we will apply the atomic lensing
323 model to some challenging systems including a core-shell nanoparticle and a high entropy alloy,
324 and will compare the predictions against the results from multislice simulations to showcase its
325 advantages and limitations.

326 *4.1. Channelling theory of atomic lensing model for spectroscopy*

327 If we assume that the electron probe wavefunction stays constant in the crystal across thickness
328 and that the scattering from each atom can be considered as being incoherent with respect to other
329 atoms, the scattering cross-section is a simple addition of the effective potentials. The scattering
330 cross-section will then increase linearly against sample thickness, noted as the linear incoherent
331 model. In reality, the electron wave function scatters dynamically giving varying contributions
332 at different depths and hence making elemental quantification difficult. In this section, we will
333 expand the atomic lensing model developed previously for ADF [24, 25] to spectroscopy with a

334 simple modification. In the atomic lensing model, we treat dynamical scattering as a superposition
 335 of individual atoms focusing the incident electrons. Here, we assume that the electron channelling
 336 effect of these individual columns alters the electron probe function and that the cross-talk of
 337 surrounding columns is negligible. By comparing the electron probe profile as a function of depth
 338 down an isolated column and an atomic column in a crystal shown in Fig. 1(c-d), the dynamical
 339 scattering is indeed largely confined to the individual columns for a sufficiently thin crystal if
 340 columns are well-separated. The dynamic switching of the electron beam between two closely
 341 spaced columns during channelling shown in [63] would break down the assumption in the atomic
 342 lensing model. Based on a simple geometric probe spreading, the column distance should be larger
 343 than the thickness times the semi-convergence angle. For the case of a 10 nm sample with a probe
 344 semi-convergence angle of 20 mrad, the spacing should be around 2 Å. One can also use a more
 345 complicated tight-binding model [63] or detailed multislice simulations to verify the channelling
 346 condition which is also elemental and wavelength dependent. Following the derivation given in
 347 [24], the focusing effect of an atomic column is given by

$$F_{col}(1 \rightarrow n) = \frac{1}{\Theta_{col,Z(n+1)}(1)} \frac{d\Theta_{col}}{dn} = \frac{\Theta_{col}(n+1) - \Theta_{col}(n)}{\Theta_{col,Z(n+1)}(1)}, \quad (11)$$

348 where $F_{col}(1 \rightarrow n)$ is the focusing effect of a column of n atoms, with atoms located at the 1st
 349 to n th position. $\Theta_{col}(n)$ is the scattering cross-section of a column consisting of n atoms. The
 350 difference between the scattering cross-section of $n+1$ atoms and n atoms is normalised by that of
 351 a single atom $\Theta_{col,Z(n+1)}(1)$ to measure the non-linear contribution from the $(n+1)$ th atom due to
 352 the lensing effect of the previous n atoms, where $Z(n+1)$ is the type of element for the $(n+1)$ th

353 atom. The lensing effect of an individual atom can be determined from the superposition principle:

$$L_Z(n) = \frac{dF_{col}}{dn} = F_{col}(1 \rightarrow n) - F_{col}(2 \rightarrow n), \quad (12)$$

354 where $L_Z(n)$ is the lensing factor of the 1st atom with atomic number Z on the $(n+1)$ th atom.
 355 Similar to optics, the lensing effect $L_Z(n)$ only depends on the relative distance away from this
 356 atomic lens, not its absolute position [24]. For instance, the lensing effect of the 1st atom on
 357 the n th atom is equal to that of the 2nd atom on the $(n+1)$ th atom (if we simply shift the absolute
 358 position while the atoms are the same). Therefore, though the scattering cross-section is non-linear
 359 against the sample thickness due to channelling, its second derivative can be linearly additive.

360 Following the superposition of lensing factors of each atom, which can be calculated from pure
 361 element libraries, we may predict the scattering cross-section of a mixed column in any order. For
 362 ADF-STEM, the predicted scattering cross-section is given by [24, 25]:

$$\Theta_{col}^{ADF}(N) = \Theta_{col}^{ADF}(N-1) + \left(1 + \sum_{n=1}^{N-1} L_{Z(n)}^{ADF}(N-n) \right) \Theta_{col,Z(N)}^{ADF}(1), \quad (13)$$

363 where $Z(n)$ is the atomic number of the n th atom in a mixed column. The lensing factor $L_Z(n)$
 364 of each atom of a column alters the electron probe function, yielding a non-linear response due
 365 to channelling, which is summed to predict the focusing effect for the next atom in sequence.
 366 The resulting scattering cross-section $\Theta_{col}^{ADF}(N)$ is predicted for a mixed column at the depth of N
 367 atoms.

368 For spectroscopy being an incoherent imaging technique, the scattering cross-section for each
 369 element can be written as:

$$\Theta_{col}^{Spec}(N, Z(N)) = \Theta_{col}^{Spec}(N-1, Z(N)) + \left(1 + \sum_{n=1}^{N-1} L_{Z(n)}^{Spec}(N-n) \right) \Theta_{col}^{Spec}(1, Z(N)), \quad (14)$$

370 where $\Theta_{col}^{Spec}(N, Z(N))$ is the scattering cross-section matrix of a mixed column with prediction
371 value at the depth of N atoms and element with atomic number $Z(N)$. Note that the atomic number
372 $Z(N)$ is a function of depth and encodes the ordering and number of atoms in a column. The
373 spectroscopy scattering cross-section matrix $\Theta_{col}^{Spec}(N, Z(N))$ is calculated in a step-wise manner,
374 with rows representing the depth and columns representing different elements. For instance, the
375 scattering cross-sections at the N th row are derived from the $(N-1)$ th row with the increment of
376 cross-section of the element with atomic number $Z(N)$ following the lensing rule. In practice, this
377 requires simulations of the EDX signals for each element to predict the EDX of mixed columns.
378 This will be applied in the Au@Pt core-shell nanoparticle case in Section 4.3.

379 Since there is a strong linear dependence when comparing ADF to EDX cross-sections as
380 examined in Section 3, we can also make EDX predictions from ADF:

$$\Theta_{col}^{Spec}(N, Z(N)) = \Theta_{col}^{Spec}(N-1, Z(N)) + \left(1 + \sum_{n=1}^{N-1} L_{Z(n)}^{ADF}(N-n) * K(Z(N)) \right) \Theta_{col}^{Spec}(1, Z(N)), \quad (15)$$

381 where $L_{Z(n)}^{ADF}(N-n)$ is the lensing factor resulting from ADF libraries of pure elements and $K(Z(N))$
382 is the slope of the ADF-EDX linear dependence for the element of interest $Z(N)$. To test Eq. 15,
383 we calculate the full ADF library at each thickness and EDX library at a finite number of thick-
384 nesses to retrieve the ADF-EDX slope using frozen phonon calculations for the high entropy alloy
385 case in Section 4.3.

386 4.2. Computational complexity and accuracy

387 A major challenge for the spectroscopy quantification of complex nanostructures is to consider
388 the channelling effect in mixed columns. The number of possible combinations in the ordering of

atoms exceeds the capability of multislice calculations. Recent developments with the PRISM algorithm provide a significant speedup alternative [64, 65], which is now available for both STEM [65, 66, 67] and EELS [68] simulations. PRISM combines the Bloch wave and multislice via the scattering matrix to alleviate the repetitive computation cost involved in each scanning probe position [64]. This is particularly attractive in the case of a large field of view. The accelerated speed is at the cost of accuracy [64, 66, 69]. However, when facing the ordering possibilities for each column multiplied by the number of potentially mixed columns, the PRISM algorithm can also be time-consuming. In contrast, the atomic lensing model is a column-by-column prediction framework [24, 25], which might be less accurate but provides a much faster albeit rough estimation. In this section, we will examine the computational cost and accuracy of the atomic lensing model against multislice calculations so that one can make a rational choice. We also include the PRISM algorithm in the computational cost benchmark as an alternative option.

Here we follow the analysis in [64] to estimate the calculation time. The computational complexity for each algorithm is given in Table 3 together with the parameters used. In contrast to the previous analysis, we also take into account the number of phonon configurations and the number of column ordering configurations, as they are indeed common multiplication factors for multislice and PRISM but not for the atomic lensing model. For the multislice algorithm with a supercell sampled by $N \times N$ pixels, each slice requires 5 forward and backward Fourier transformations (complexity: $5N \log_2 N$) together with a wave function multiplication with the potential in real space and with the Fresnel propagator in reciprocal space (complexity: $2N^2$) [64]. This complexity is amplified with (1) the number of slices H , (2) the number of probe positions P , (3) the number of phonon configurations T , and (4) the number of possible orderings O in mixed columns.

411 The PRISM algorithm only needs to perform the repetitive transmission-propagation in the mul-
412 tisllice once to construct the scattering matrix for each parallel beam sampled. The number of
413 beams needed B can be factorised by the interpolation factor f . The effect of the number of probe
414 positions P is added later, which is outside of the multislice loop (complexity: $PBN^2/4f^4$) [64].
415 However, the computational time still scales with the ordering possibilities. In contrast, the atomic
416 lensing model only needs the multislice calculations to build the pure element libraries. The fol-
417 lowing calculations to generate the scattering cross-sections for a mixed column of any ordering
418 are simple numerical operations in Eq. 13-14 and are only dependent on the number of possible
419 elements E and the number of atoms (at same order as the number of slices H) in a column. Note
420 that the scattering cross-section is a single value predicted for a column instead of a full image
421 simulated in multislice and PRISM. Also, note that the atomic lensing model prediction for each
422 column is treated independently. Hence the total number of orderings for a system is a summation
423 of the orderings in each column. The column-by-column approach simplifies the exploration of
424 ordering and provides a significant speedup in predictions, which however is also the major source
425 of error as we can see later in the benchmark and case studies.

426 To benchmark the speed and accuracy, we tested the computation time against the number of
427 column orderings in an Al-Ag binary alloy crystal with a random ordering and a supercell made
428 of $8 \times 8 \times 20$ FCC unitcells. We used the MULTEM software [60] for the multislice simulation
429 with the parameters in Table 1 and the abTEM software [67] for the PRISM algorithm with an
430 interpolation factor of 20 tested on a desktop with an Intel i7-8700K CPU and a Nvidia RTX 1080
431 GPU. We only benchmarked the ADF computation time, because PRISM does not have the EDX
432 capability yet and our prototype EDX multislice is not optimised for GPU (to be implemented).

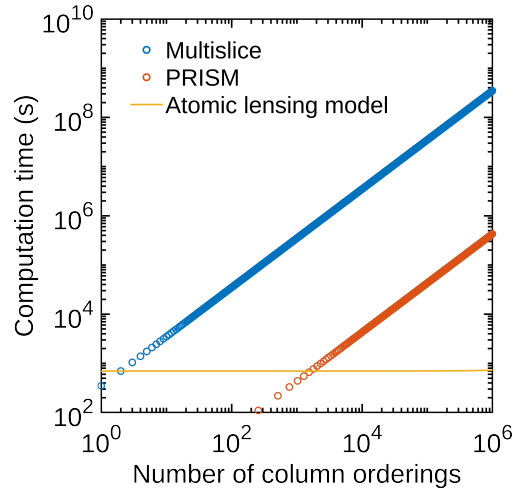


Figure 5: Comparing the computation time for the multislice simulation, the PRISM simulation, and the atomic lensing model for predicting the scattering cross-section against the number of ordering configurations in an Al-Ag binary alloy crystal.

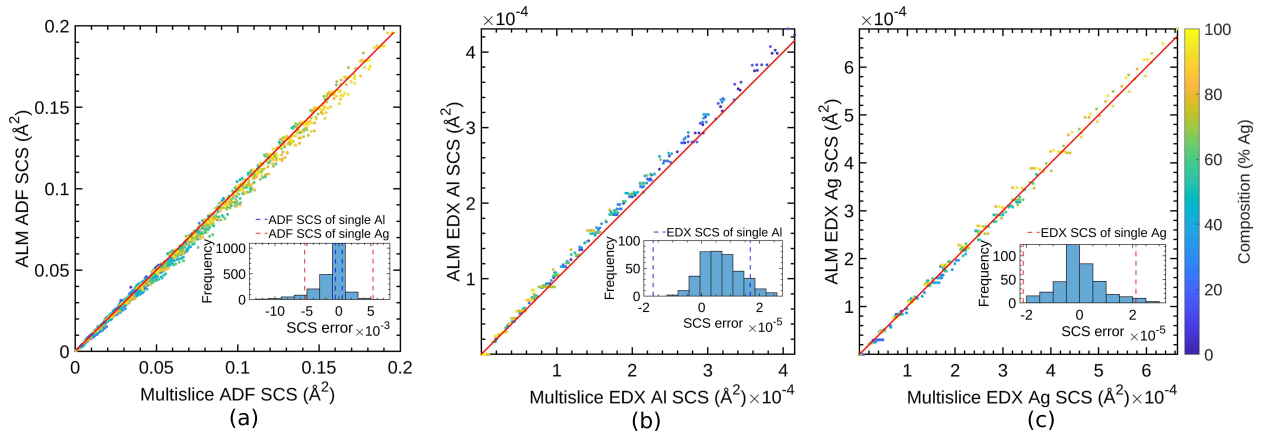


Figure 6: Multislice simulated against the atomic lensing model predicted scattering cross-sections for (a) ADF, (b) EDX Al and (c) EDX Ag, with a red line indicating the perfect predictions. The histograms of the absolute errors are given in the insets.

Table 3: Computational complexity of the multislice simulation, the PRISM simulation and the atomic lensing model.

Algorithm	Computational complexity
Multislice	$OTHP[5N\log_2N + 2N^2]$
PRISM	$OT[\frac{HB}{f^2}[5N\log_2N + 2N^2] + \frac{PBN^2}{4f^4}]$
Atomic lensing model	$ETHP[5N\log_2N + 2N^2] + OHE$
Parameter	definition
O	number of ordering configurations
T	number of phonon configurations
H	number of slices
P	number of probe positions
N	side length (in pixels) for supercell sampling
B	number of beams
f	interpolation factor
E	number of elements in the system

433 The EDX computational time will be on a similar scale as ADF once optimised. As shown in
434 Fig. 5, a new multislice simulation is needed for each different ordering, hence its computational
435 time is extrapolated linearly against the number of column orderings to be computed, with each
436 column taking ~ 350 s. The PRISM algorithm outputs all the columns in the input supercell si-
437 multaneously thanks to the shared scattering matrix, which is much faster per column (~ 110 s for
438 256 columns) but still has a linear scaling against the number of column orderings. In contrast,
439 the most time-consuming part of the atomic lensing model is the library generation via multislice

440 simulations which scales with the number of elements in the system. The prediction, however, is
441 as fast as $29 \pm 5 \mu\text{s}$ per column showing an almost constant behaviour in the log-log plot in Fig. 5.
442 The atomic lensing model is the only feasible approach that can explore all the ordering possi-
443 bilities for a 20-atom-thick binary alloy column, taking ~ 30 s to loop over 1 million orderings.
444 Instead of making new predictions again for another column, one can simply adopt the existing
445 predictions as a look-up table for different thicknesses and orderings. Storage of such a database
446 increases linearly with the ordering configurations which will eventually become challenging for
447 thick samples. For example in a binary alloy system, storing the EDX cross-sections of 2 elements
448 for 20 atoms (with $\sim 10^6$ configurations) will take 8 Mb for storage, this would increase to 8 Tb
449 for 40 atoms (with $\sim 10^{12}$ configurations).

450 To benchmark the accuracy, we sampled the Al-Ag alloy composition in the range of 1-99%
451 Ag with 1% interval for ADF and 5-95% Ag with 5% interval for EDX with different ordering in
452 all columns for each composition. In each case, one column was selected for the probe to scan
453 over the corresponding Voronoi cell and measure its scattering cross-section. Fig. 6 shows the
454 atomic lensing model predicted ADF and EDX scattering cross-sections against those quantified
455 from multislice for different thicknesses and compositions (indicated by colors). We can see that
456 most of the predicted values are in close agreement with simulations where the red line indicates
457 a perfect match. The histograms of the absolute errors, defined as the difference between the
458 predicted and simulated values are shown in the insets of Fig. 6. From these histograms, it follows
459 that most ($\sim 95\%$) of the prediction errors are within the scattering cross-section of a single Al or
460 Ag atom – indicating the mis-prediction is less than ± 1 atom. We do not compare the PRISM
461 accuracy further in this paper as it has been discussed in several studies [64, 66, 69], which is

462 highly dependent on the interpolation scheme. The interpolation factor of 20 used in this study
463 corresponds to $\sim 10\%$ error in PRISM as shown in [64].

464 4.3. Case studies: core-shell nanoparticle and high entropy alloy

465 The atomic lensing model allows for a fast generation of scattering cross-sections with the
466 ordering of elements taken into account under the channelling condition. In this section, we will
467 demonstrate the accuracy and limitation of the atomic lensing model in predicting the ADF-EDX
468 scattering cross-sections of mixed columns. The results will be compared against multislice sim-
469 ulations and the linear model. Note that the linear incoherent model here refers to cross-sections
470 increasing linearly with the number of atoms, which is different from the linear dependence be-
471 tween ADF-EDX signals.

472 One cannot readily distinguish the presence between Pt and Au based on an ADF image since
473 their atomic numbers only differ by 1. However, we can separate them unambiguously based on
474 their spectroscopy signals as shown in Fig. 7 for a core-shell Au-Pt nanorod. To quantify the im-
475 ages, both the ADF and EDX scattering cross-sections are extracted from the simulations using
476 Voronoi cell integration, which agree reasonably well with the atomic lensing model predictions
477 (relative error ADF $< 5\%$, EDX $< 10\%$). Columns close the vacuum and at the core-shell inter-
478 face result in the largest deviations. Those results can be understood from the fact that the atomic
479 lensing model is based on pure elemental libraries, which unavoidably treats the contributions of
480 surrounding columns as pure elements thus deviating from reality. In contrast, the linear model
481 significantly underpredicts the signals since electron channelling is ignored. We noticed that the
482 nanoparticle can undergo surface relaxation leading to misalignment of atomic columns and hence

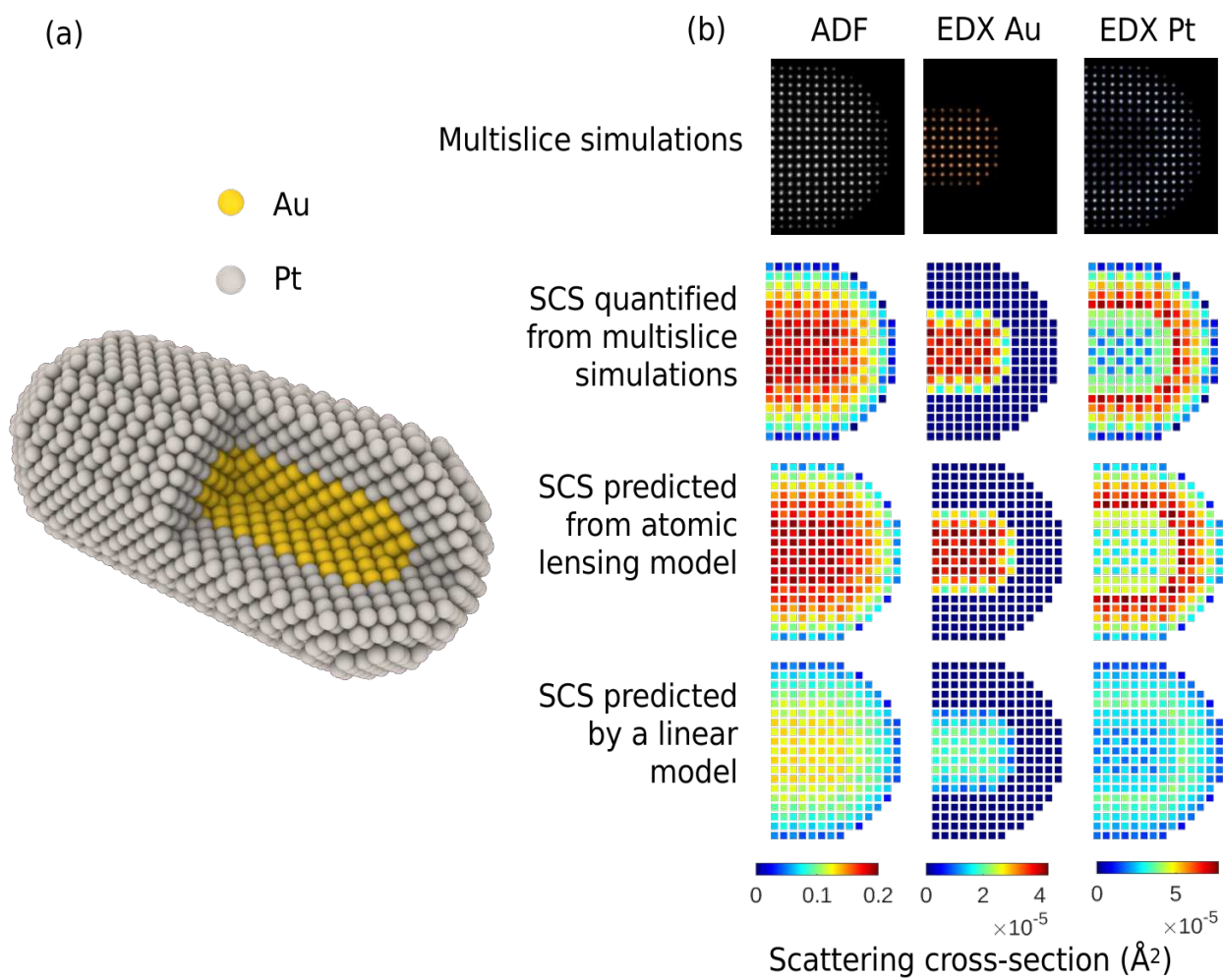


Figure 7: (a) Atomic model of the Au@Pt core-shell nanoparticle. (b) Comparison of the simulated multislice quantified, atomic lensing model (ALM), and linear incoherent model predicted ADF-EDX scattering cross-sections (SCS). The simulation parameters are given in Table 1-2.

483 causing a larger error for the atomic lensing model which is based on perfect crystal libraries. In
 484 addition, microscopy experiments are often under limited doses thus affecting the measurement
 485 accuracy while simulations shown here are at infinite doses. Readers can find our further investi-
 486 gation of the atomic lensing model for combined ADF-EDX atom counting with limited dose and
 487 simulated particle relaxation in [70].

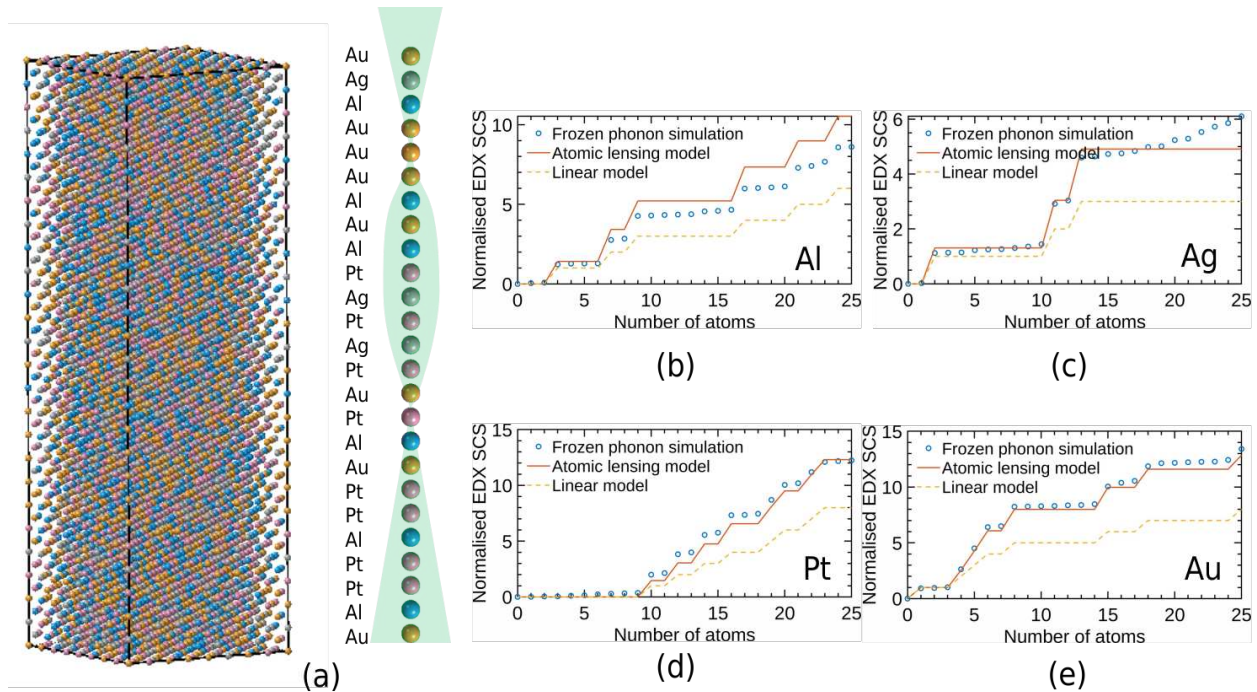


Figure 8: (a) A 3D model of the Al-Ag-Pt-Au high entropy alloy slab with 25 atoms in each atomic column along the electron beam direction in an FCC [0 0 1] orientation. The ordering of a particular column is given, which is used for comparing the simulated values and predictions from the atomic lensing model and linear model. The normalised EDX scattering cross-sections of this column are plotted as a function of the number of atoms for (b) Al, (c) Ag, (d) Pt, and (e) Au respectively. The simulation parameters are given in Table 1-2.

488 To evaluate the atomic lensing model in nano-materials containing both heavy and light el-
 489 ements which result in complicated electron channelling, we randomly substitute an Au crystal

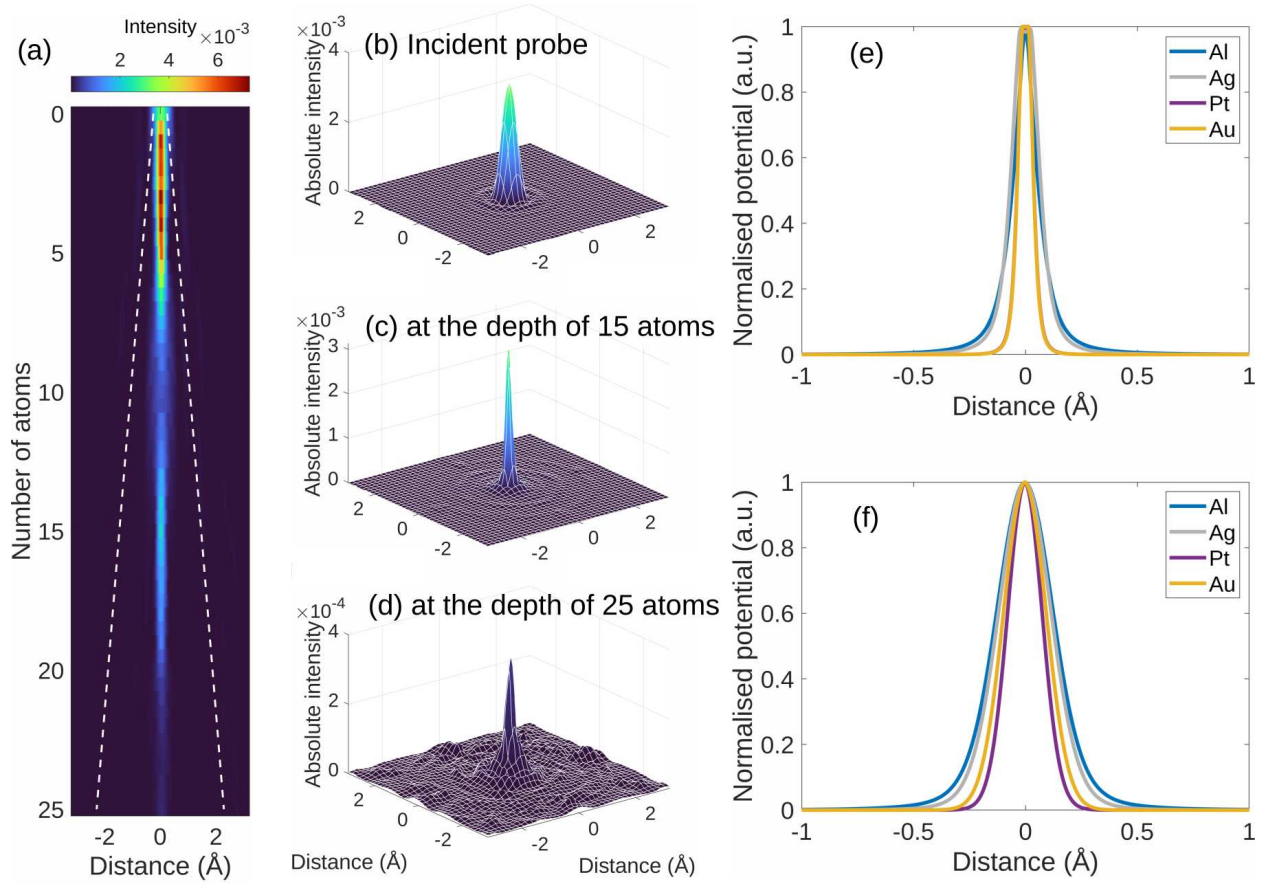


Figure 9: visualisation of beam broadening and the widths of ionisation potentials. (a) Probe profile (shown on a square root scale) as a function of depth with a probe placed on the high entropy alloy column of interest in Fig. 8. The dashed lines indicate the geometric probe spreading. The corresponding real space intensity maps are given for (b) the incident probe (c) at the depth of 15 atoms and (d) at the depth of 25 atoms. Note that 25 atoms in depth correspond to around 10 nm. Plots of normalised ionisation potentials for Al(1s)-Ag(2p)-Pt(2p)-Au(2p) core orbitals before and after thermal smearing are given in (e-f), using the inelastic scattering factor from [40]. The potentials are normalised against their maximum values for better visualisation of the delocalisation. The simulation parameters are given in Table 1-2.

490 with Al, Ag, and Pt, each taking 25% of the sites of the full lattice, to form a high entropy alloy.
491 The full 3D crystal model and the ordering for a particular column under investigation are given
492 in Fig. 8(a). In Fig. 8(b-e), it is shown that the overall channelling behavior is well captured by the
493 atomic lensing model for heavy elements but there are deviations for light elements. Specifically,
494 Fig. 8(a) shows that the Al scattering cross-section is overestimated with increasing thickness. In
495 addition, Fig. 8(c) shows an increasing Ag scattering cross-section against sample thickness, while
496 there is no Ag in the ordering of this column beyond a depth of 13 atoms. As indicated by Eq. 9,
497 the EDX signals are determined by the real space overlap of electron intensity at a given depth
498 and the ionisation potentials of the corresponding elemental core-shell orbitals. These deviations
499 result from the beam spreading to neighbouring atoms and the delocalization of their ionisation
500 potentials.

501 Since the spatial spread of the electron beam varies with increasing thickness, both due to the
502 geometry spread of a cone-shaped beam and the scattering by the atoms, the EDX contribution
503 from neighbouring atoms will become important and column-by-column analysis shall eventually
504 break down. Fig. 9(a) shows the probe profile as a function of depth with the probe placed at
505 the high entropy column of interests. Fig. 9(b-d) shows the real space probe intensity for the
506 incident probe alongside the probe at the depth of 15 and 25 atoms. At the depth of 15 atoms,
507 the channelling effect maintains the probe peak intensity at the same order of magnitude as the
508 incident beam. However, the peak intensity drops by a factor of 10 at a depth of 25 atoms, with
509 the ripples of the electron density distribution also weakly peaked at the surrounding columns
510 $\sim 2\text{\AA}$ apart. The Ag signal increment with no Ag in the later sequence of the column is a clear
511 result of the beam spreading exciting signals from neighbouring columns. The EDX contribution

512 of neighbouring columns in thick SrTiO₃ samples at fixed probe positions was examined in [71],
513 suggesting a careful balance of signal-to-noise ratio and delocalisation of EDX with an increasing
514 sample thickness for the column-by-column analysis. We also refer interested readers to [37] for
515 an example of the quantification of a heterophase interface and [72] for a column-by-column EELS
516 quantification correction method.

517 Concerning delocalization, we can consider (a) inherent ionisation potential, and (b) thermal
518 vibrations of atoms. The inherent width of the EDX potential for a light element is larger due to the
519 loosely bounded core-electron as compared to heavy elements, shown in Fig. 9(e). In addition, be-
520 cause of its lighter weight and weaker interatomic bonding, the light elements are displaced further
521 away from their equilibrium positions given in Table 2, resulting in an even broader potential after
522 thermal smearing shown in Fig. 9(f). Based on Eq. 9, a broad effective potential of light elements
523 leads to a low EDX yield for a given electron probe, which could well be the case for Al signals.
524 In general, those deviations originate from the difference between the channelling approximation
525 based on the atomic lensing model with pure libraries and real scattering in a mixed column with
526 surrounding columns, which is almost unavoidable with the underpinning independent column ap-
527 proximation. For future studies, we are currently working on improving the atomic lensing model
528 by considering beam spread and neighbouring columns. We will also explore the possibilities of
529 a "hybrid" strategy for the quantification of mixed columns: i.e. using the atomic lensing model
530 to provide good starting predictions, which can then be further refined using multislice or PRISM
531 calculations.

532 **5. Conclusions**

533 In this manuscript, we proposed a method for a fast prediction of the ADF-EDX scattering
534 cross-sections under channelling conditions. EDX signals are fully incoherent following the in-
535 elastic scattering theory. For ADF with a sufficiently high inner collection angle, the incoherent
536 phonon scattered electrons dominate the contrast while the elastically scattered electrons also be-
537 come longitudinally incoherent, thus establishing a linear dependence between ADF and EDX
538 signals against sample thickness. We examined the validity of this linear dependence as a function
539 of ADF collection angles under different microscope conditions. In addition, this also maps the
540 ADF longitudinal incoherency.

541 Since both the ADF and EDX are incoherent imaging modes, we expanded the atomic lensing
542 model previously developed for ADF to EDX, which could also be applicable for EELS with a
543 large collection angle. The model takes the 3D ordering of the atomic column into account by
544 describing the dynamic diffraction as a superposition of the lensing effects of individual atoms fo-
545 cusing the incident electrons. The speed and accuracy of the atomic lensing model were compared
546 against multislice and PRISM algorithms. We demonstrated that this model can reliably predict
547 EDX values for a Pt@Ag core-shell nanoparticle and for an Al-Ag-Pt-Au high entropy alloy up
548 to 25 atoms (10 nm). Beyond this thickness, the contribution of neighbouring columns becomes
549 significant. This method opens opportunities to quantify atomic resolution EDX and to explore
550 the enormous amount of ordering possibilities of heterogeneous materials with multiple elements.

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557 muSTEM calculations.

558 **References**

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