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INVESTMENT IN A NEW TECHNOLOGY UNDER TECHNOLOGICAL UNCERTAINTY: THE VALUE OF THE OPTION TO ABANDON

Compernolle T.*a, Van Passel S.a, Huisman K.b,c, Kort P.b,d

^aHasselt University, Centre for Environmental Sciences, Agoralaan, Building D, B-3590 Diepenbeek, Belgium

^bTilburg University, Department of Econometrics & Operations Research and CentER, Warandelaan 2, 5037 Tilburg, The Netherlands

^cASML, De Run 6501, 5504 Veldhoven, The Netherlands

^dUniversity of Antwerp, Department of Economics, Prinsstraat 13, 2000 Antwerp, Belgium

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ABSTRACT

To economically evaluate investments in clean technologies under technological uncertainty, it is widely accepted that the real options theory better deals with issues like uncertainty and the flexibility of the decision process than traditional net present value methods. Contrary to the standard real options effect, this study demonstrates that the option value stimulates investment. Furthermore, this real options analysis not only considers the *ex ante* decision analysis of the investment in a new technology under uncertainty, it also allows for an *ex post* evaluation of the investment. Based on a case study regarding the adoption of an innovative groundwater remediation strategy, it is demonstrated that when the option to abandon the innovative technology is taken into account, the decision maker decides to invest in this technology, while at the same time it determines an optimal timing to abandon the technology if its operation proves to be inefficient. To reduce uncertainty about the effectiveness of

^{*} Corresponding author, (e) tine.compernolle@uhasselt.be, (t) +32 11 26 87 49

groundwater remediation technologies, samples are taken. Our analysis shows that when the initial belief in an effective innovative technology is low, it is important that these samples provide correct information in order to justify the adoption of the innovative technology.

1. INTRODUCTION

In this introduction, we first review the parts of the real options theory that are relevant for our study. We focus on technology adoption and the role of technical uncertainty. Because groundwater remediation is the case study subject, we also introduce the groundwater remediation techniques considered. We further explain how this case study relates to previously performed studies.

Real options theory

Concerning the economic evaluation of investment projects that aim to achieve sustainable development, one has become more aware that discounted cash flow (DCF) methods are inadequate to deal with issues like uncertainty, the irreversibility of an investment decision, and the flexibility of the decision process (Boomsma et al. 2012; Diederen et al. 2003; Fernandes et al. 2011). It is widely demonstrated that the real options approach gives a better insight into the management of natural resources (Guthrie and Kumareswaran 2009; Murillas and Chamorro 2006) and into the adoption of pollution control and renewable energy systems, including the evaluation of policy support (Fuss and Szolgayová 2010; Heydari et al. 2012; Reuter et al. 2012; Wirl 2006).

The literature on technology adoption mostly considers the *ex ante* adoption problem under technological uncertainty taking into account the value of waiting (Hart 2009). Dosi and Moretto (1997) explore the relationships between the design of public incentives and a firm

that faces uncertain benefits from the abandonment of a polluting technology. These authors point out that the green investments' irreversibility and uncertainty about related benefits might delay environmental innovations. Farzin *et al.* (1998) investigate the optimal timing of technology adoption by a competitive firm that faces a stochastic innovation process with uncertainties about the speed of the arrival and the degree of improvement of new technologies. They demonstrate that investments in new technologies slow down when firms are already at the forefront of technological efficiency. When the pace at which technological improvements arrive is fast, the optimal timing of adoption is delayed. Also Fuss and Szolgayová (2010) find that uncertainty associated with technological progress leads to a postponement of investment, which can enforce the lock-in of currently applied systems.

Kline (2001) states that policy response to lock-in should pursue the encouragement of experimentation with alternative technical and institutional approaches to environmental management. A broad range of technology options should be surveyed without considering what the market outcomes might produce. Both learning by doing and learning by using can result in increased technology improvements and the growth of technology demand (Mukoyama 2006; Rosenberg 1982). Rosenberg (1982) emphasizes the importance of learning through the utilization of the new technology. An early experience with a new technology not only leads to a better understanding of the relationship between specific design characteristics and performance, it can also result in new practices that increase the productivity of the project. Also Grübler and Messner (1998) state that technological change is a result from R&D, technology demonstration, and investments. Without short-term investments, long-term technology improvements will not materialize.

This real options analysis not only considers the *ex ante* decision analysis of the investment in a new technology under uncertainty, it also allows for an *ex post* evaluation of the investment. The decision maker includes the possibility to evaluate the performance of the new technology into the decision making process. Based on a case study, it is demonstrated that without this option, the decision maker does not invest in the new technology. If the option to redirect the decision is taken into account, the decision maker invests in the new technology and the optimal timing to stop the operation if the new technology proves to be inefficient is determined.

Groundwater remediation

Groundwater is an essential resource that should be protected and managed properly. However, fuel storage tank leakages, accidental spills, and the excessive use of pesticides are only a few of the many sources contaminating groundwater. If groundwater contamination occurs, it is important to remediate or at least contain the contamination in order to prevent transport to lakes and rivers by natural discharge (Hardisty and Özdemiroglu 2005). When the market for groundwater remediation is considered, there is one technique dominating: pump & treat (USEPA 2010). Pump & treat involves the extraction of contaminated groundwater which is then treated above ground (USEPA 1996). More gentle remediation techniques like bioremediation are not widely accepted and only applied to a limited extent. Stakeholders and policy makers who are not familiar with these kind of remediation techniques, still need to be convinced of its merits (Compernolle et al. 2011; Vangronsveld et al. 2009).

Bioremediation involves the extraction of groundwater, but unlike the pump & treat technology, the extracted groundwater is enriched with nutrients and injected in a recharge well. The nutrients then activate the bacteria present in the groundwater, which results in the

degradation of the contamination (Juwarkar et al. 2010). The technological uncertainty addressed in this paper stems from the variability inherent to bioremediation processes. Factors such as low temperature, anaerobic conditions, low levels of nutrients and cosubstrates, the presence of toxic substances, and the physiological potential of microorganisms can limit the efficiency of microbial degradation (Megharaj et al. 2011). It relates to the physical difficulty to achieve certain goals: it is not known beforehand how much time, effort and materials will ultimately be required to meet the objectives set.

This kind of uncertainty does not induce a value to wait, it can only be resolved by undertaking the project (Pindyck 1993). One then observes how actual costs, or the actual efficiency of the technology in our case, unfold as the project proceeds. In this study, a firm has the opportunity to invest in a bioremediation technology of which the contaminant mass removal efficiency is uncertain. After having invested, the firm faces the decision to continue the operation or to redirect it and adopt pump & treat, a technology it is more familiar with. It is not known beforehand how effective bioremediation will be. However, from the moment bioremediation is started, the contaminant mass removal efficiency is evaluated by taking groundwater quality samples, indicating bioremediation either to perform good or bad.

This type of problem closely relates to the studies performed by Jensen (1982) and Thijssen *et al.* (2004). Jensen (1982) describes a decision problem in which an innovation is introduced but the firm does not know whether the adoption will be profitable or not. By waiting and gathering information, this uncertainty can be resolved. This decision problem is formalized as an optimal stopping problem in which the firm can either invest, *i.e.* adopt the innovative project and receive the expected return, or wait, learn from the observations and receive the expected value of this information. The firm starts with an initial belief concerning the

profitability of the innovation, which it updates each time it receives new information. The firm's learning behavior is assumed to be Bayesian: the belief is a conditional probability based on past information. While Jensen (1982) only showed existence of a critical value of the belief in a good project at which investing is optimal, Thijssen *et al.* (2004) extend this study and develop a framework in which an explicit expression is provided for this critical value.

Our study uses the framework of Thijssen *et al.* (2004) to find the critical value of the belief in bioremediation at which the firm decides to stop its operation and adopt pump & treat. Unlike the study of Thijssen *et al.* (2004), the firm does not receive information if it waits and therefore has to decide whether or not to adopt bioremediation (of which the outcome is uncertain) immediately. This means that an initial value associated with the innovative project is included in the economic framework. If the firm decides to invest, it gathers information on the effectiveness (the revenues) of the bioremediation technology by evaluating its performance based on groundwater samples. Using this information, the firm decides to continue or replace it by pump & treat of which the outcome is certain. Thijssen *et al.* (2004) model the arrival of information, *i.e.* market signals, via a Poisson process. Also this is different from our study. For our decision problem, samples being taken at fixed moments in time are the source of information. The quality of the signals, *i.e.* the probability with which the sample reflects the true state of the world, is modeled as a binomially distributed random variable.

The paper is organized as follows. The specific case study and the scenarios considered are described in Section 2. Two scenarios are considered: with and without the option to abandon. In Section 3 the theoretical models for these scenarios are outlined and it is shown that

without the option to abandon the remediation technology, the firm will decide to adopt pump & treat, the remediation strategy of which the contaminant mass removal efficiency is certain. Then, the model that includes the option to abandon is introduced. By taking groundwater samples during the remediation process, information on the performance of the new technology becomes available. Our analysis shows how this information is integrated in the economic valuation of the new technology and how the firm defines an optimal timing to abandon the bioremediation technology if it does not perform in a sufficient manner. If the option to abandon is included in the decision analysis, then pump & treat is not necessarily the preferred remediation technology. We find that when the initial belief in an effective innovative remediation strategy is low, it is important that the groundwater samples taken provide correct information in order to justify the adoption of the innovative technology. In Section 4, the robustness of the case study results is evaluated. In a final section, the results are discussed.

2. CASE STUDY

The case study developed is largely based on the characteristics of a contaminated area at an industrial site (Ineos ChlorVynils) for which two remediation alternatives are available for clean-up. The first groundwater remediation strategy is the application of a pump & treat system that results in a relatively large removal of contaminated mass but that is also relatively expensive. The second strategy is a bioremediation strategy which is less expensive but the quantity of contaminant mass removed is smaller and uncertain. Based on this case study, it is shown that when the option to abandon the bioremediation strategy is considered, the firm's *ex ante* decision is to invest in the bioremediation strategy. The economic model also allows for an *ex post* evaluation that based on groundwater samples taken determines an optimal timing to switch to pump & treat if the operation of the bioremediation strategy

proves to be inefficient. If the option to abandon is not included, the firm will decide to adopt pump & treat.

To show how the theoretical real option model can be applied to a real-life case study, a simplification of the contaminated area is made. Although the groundwater body considered is fictive, its characteristics are representative for groundwater bodies in Belgium, and the design of the remediation alternatives and cost information are based on the information provided by the firm. It is assumed that an area of 50 000m² is contaminated over a depth of 25m with a concentration of 100 mg per liter, as shown in Figure 1. This corresponds to 95 ton contaminated mass initially present (assuming a retardation factor¹ of 2 and a porosity² of 0.38). In order to evaluate the effectiveness of the different remediation strategies, a groundwater flow and transport model is used to simulate both pump & treat and bioremediation. This model is developed using the code MOCDENS3D in which MODFLOW is integrated to calculate groundwater flow (Lebbe and Oude Essink 1999; Vandenbohede 2010). Table 1 gives an overview of the hydrogeological input parameters applied. The decision maker can decide to adopt pump & treat (strategy 1) or to apply bioremediation (strategy 2).

Strategy 1. With respect to the P&T remediation strategy, one extraction well with a screen length of 25 m is put in place in the centre of the contaminated area, fully penetrating the aquifer. This is presented by the dashed circle in Figure 2. The extracted groundwater is assumed to be treated above ground using granular activated carbon.

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¹ The retardation factor describes how many times faster the groundwater is moving relative to the contaminant being sorbed.

² Porosity is the void volume relative to the total volume of the subsurface considered.

Strategy 2. With respect to the bioremediation strategy, a combination of P&T and bioremediation is considered. A configuration of one recharge well (RW2) and a T-bioremediation cell, as presented in Figure 2, is applied. The T- bioremediation cell consists of 3 extraction wells (EW1-3), one central extraction well (CEW) and one recharge well (RW1). The bioremediation T-cell operates as follows. In a first stage, the EW1- EW3 and RW1 wells are used to distribute nutrients around the central extraction well (CEW). Groundwater is extracted by the three extraction wells (EW1-3). The extracted groundwater is enriched with nutrients and injected in the recharge well (RW1). The nutrients then activate the bacteria present in the groundwater, which results in the degradation of the contamination. The central extraction well does not yet operate. Bioremediation adopted during the first stage is followed by a flushing system. In a second stage, the central extraction well extracts enriched and decontaminated groundwater, which is then injected in recharge well 2 (RW2) to induce an increased groundwater flow and flush the contaminated site.

The simulation of the bioremediation strategy only considers the hydrogeological aspects. Biological growth and degradation processes are not included in the simulation. Hence, in order to evaluate the effectiveness of the bioremediation strategy, only the second stage is simulated. This involves the extraction at the central extraction well and the injection of the enriched groundwater in recharge well 2. The biodegradation rate can vary between 0% and 100%. As the probability density function for this parameter is unknown, we decided to use an upper and lower limit for the biodegradation rate. The upper limit is determined by the average quantity of contaminant mass removed for a biodegradation that varies between 50% and 100%. The lower limit is determined by the average quantity of contaminant mass removed for a biodegradation rate that varies between 0% and 50%.

Using the groundwater model, it is determined that the lower limit of the annual quantity of contaminant mass removed equals 2 656 kg. The upper limit equals 3 920 kg. For the pump & treat strategy, the annual quantity of mass removed is determined at 4 712 kg. The valuation of the quantity of contaminated mass removed is based on a property value of €23 m⁻². For a contamination rate of 0.019 ton m⁻², one ton of contaminated mass removed is valued at €12 105. The costs of the bioremediation and pump & treat remediation strategies are determined using information provided by the firm Ineos ChlorVynils³ that actually had to choose between these remediation technologies. Implementing the bioremediation strategy results in an annual cost of €30 624. The pump & treat system costs €49 112 annually. Table 2 gives a detailed overview of the annual cost and the investment costs considered. Table 3 presents the results of calculating the annual net cash flows.

The decision whether or not to invest in bioremediation is determined taking two different scenarios into consideration. In a first calculation, the value of the bioremediation strategy is determined without the option to abandon. Secondly, the option to abandon the bioremediation strategy is integrated in the investment decision. These two calculations represent two different decision processes. Figure 3 graphically presents the difference between these two processes. The squares are decision nodes. The circle is an uncertainty node, *i.e.* it represents the different contingencies that can occur.

3. MODEL

Initially, the firm has a prior belief about bioremediation being efficient or inefficient. The *ex ante* probability of the technology being efficient is given by

$$\mathbb{P}(H) = p_0. \tag{1}$$

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³ The source documents in which these cost data are listed can be made available on request

In Table 4, the first row lists the probabilities in case of an efficient technology (H) and the second row in case of an inefficient technology (L). Hence, H and L represent the true state of the world in case of an efficient and inefficient technology, respectively. At determined moments in time, the firm evaluates the technological efficiency by taking a sample indicating the technology to be efficient (h-sample) or inefficient (l-sample). It is assumed that a correct sample always occurs with probability $\lambda > \frac{1}{2}$ (see Table 4).

The samples are taken at fixed moments in time. The type of the sample is modeled as a binomially distributed random variable, indicating bioremediation either to be good or bad. Denoting the number of signals that have arrived before time t by n(t), $t \ge 0$, and denoting the number of h-signals by g, the dynamics of g is given by udn(t) = u, with dn(t) = 1,

$$u = \begin{cases} 1 \text{ with probability } \lambda \text{ if H and } 1 - \lambda \text{ if L,} \\ 0 \text{ with probability } 1 - \lambda \text{ if H and } \lambda \text{ if L,} \end{cases}$$

and g(0) = 0.

It is assumed that the firm knows the value of λ . The belief that revenues are high, *i.e.* that the bioremediation technology is efficient, given the number of samples n and the number of h-samples, $g \le n$, is denoted by p(n,g). The conditional expected payoff of bioremediation can be written as

$$\mathbb{E}(U|n,g) = p(n,g)U_{Biorem}^H + (1 - p(n,g))U_{Biorem}^L - C_{Biorem}, \tag{2}$$

in which U^H_{Biorem} and U^L_{Biorem} represent the incoming cash flow associated with a high and low technological efficiency, respectively. C_{Biorem} is the outgoing cash flow that comprises the operational costs. The firm stops bioremediation and adopts pump & treat as soon as the belief in bioremediation being efficient, reaches some critical level. To calculate this critical level, first p(n,g) should be determined. Define k=g-(n-g)=2g-n, the number of good

samples in excess of bad samples, and $\zeta = (1 - p_0)/p_0$, the unconditional odds of the technology being inefficient. By using Bayes' rule we obtain that

$$p(n,g) = \frac{\mathbb{P}(n,g|H)\mathbb{P}(H)}{\mathbb{P}(n,g|H)\mathbb{P}(H) + \mathbb{P}(n,g|L)\mathbb{P}(L)} = \frac{\lambda^k}{\lambda^k + \zeta(1-\lambda)^k} \equiv p(k). \tag{3}$$

The critical level of k where the firm is indifferent between continuing bioremediation and adopting pump & treat is denoted by $k^* \in \mathbb{R}$. At any arrival of an h-sample, k increases with unity and at any arrival of an l-sample, k decreases with unity. Hence, enough l-samples must arrive to reach the critical level. The critical level of the conditional belief in an efficient bioremediation technology is denoted by $p^* = p(k^*)$.

Without the option to abandon

Consider a firm that has to decide whether to adopt bioremediation or pump & treat to remediate the contaminated site. Assume that the firm does not consider the possibility to abandon the bioremediation strategy. If the firm adopts bioremediation, the contaminant mass removal efficiency is uncertain, it can be either low or high. If the firm decides to install a pump & treat system, then it knows exactly how much contaminant mass is removed. Since additional information about the effectiveness of bioremediation is only obtained when it is in use, the firm makes it decision at the initial point in time. The value of the bioremediation technology is calculated as follows:

$$\mathbb{E}(U|n,g) = p_0 \sum_{t=0}^{\infty} \frac{U_{Biorem}^H - C_{biorem}}{(1+r)^t} + (1-p_0) \sum_{t=0}^{\infty} \frac{U_{Biorem}^L - C_{biorem}}{(1+r)^t} - I_{biorem}$$

=
$$p_0 \frac{(1+r)}{r} (U_{Biorem}^H - C_{biorem}) + (1-p_0) \frac{(1+r)}{r} (U_{Biorem}^L - C_{biorem}) - I_{biorem}$$

= -€18 263, (4)

with
$$p_0 = 0.25$$
, $U_{Biorem}^H = \text{€}47 \text{ 453}$; $U_{Biorem}^L = \text{€}32 \text{ 151}$; $C_{biorem} = \text{€}30 \text{ 624}$; $I_{biorem} = \text{€}128 \text{ 667}$; $r = 0.05$.

The parameter U_{Biorem}^H represents the incoming cash flows given a high efficient bioremediation project, U_{Biorem}^L represents the incoming cash flows given a low efficient bioremediation project, C_{Biorem} represents the outgoing cash flows of the bioremediation project, and I_{Biorem} represents the investment cost of the bioremediation project. With the exception of parameters r and p_0 , all other parameter values are presented in Tables 2 and 3. The parameter r is the discount rate. Because the value of p_0 is unknown, this parameter value is varied in a sensitivity analysis. The NPV of the investment in pump & treat equals

$$NPV_{P\&T} = \sum_{t=0}^{\infty} \frac{U_{p\&t} - C_{p\&t}}{(1+r)^t} - I_{p\&t} = (U_{p\&t} - C_{p\&t}) * \frac{1+r}{r} - I_{P\&T} = £24 498,$$
 (5)

with $U_{p\&t} = \epsilon 57~045; C_{p\&t} = \epsilon 49~112; I_{p\&t} = \epsilon 142~084; \ r = 0.05.$

The annual revenue, the annual cost and the investment cost of the P&T remediation strategy are represented by $U_{p\&t}$, $C_{p\&t}$, and $I_{p\&t}$, respectively. Also these parameter values are presented in Table 2 and Table 3. Based on the results of equations (4) and (5), and thus assuming for the moment that $p_0 = 0.25$, the decision maker will decide to adopt the pump & treat technology.

Integrating the option to abandon

In this section, the firm takes into account the option to abandon when it values the bioremediation strategy. It can decide to redirect the remediation process, depending on the value of the bioremediation strategy. To determine under which conditions it is necessary to abandon the bioremediation strategy and to identify the optimal timing to change the remediation strategy, the real options theory is integrated in the decision model. Based on the samples taken, the firm updates its belief in an efficient bioremediation strategy.

Suppose that the state of the process at a particular point in time is given by k. For the moment assume that k is a continuous variable. Then there are three possibilities. First, k might by such that $k \le k^*$ and hence $p(k) \le p^*$. Then it is optimal for the firm to directly abandon bioremediation and adopt pump & treat. A second possibility is that, even after an l-sample arrives, it is not optimal to abandon the bioremediation strategy, *i.e.*, $k > k^*+1$. A third and last possibility, is that $k^* < k \le k^*+1$. The value of k is such that it is not optimal to invest in pump & treat right away. However, when the following sample is an 1-sample, it will be optimal to abandon bioremediation. The value function V for the firm then equals

$$V(k) = \begin{cases} V_1(k) & \text{if } k > k^* + 1 \\ V_2(k) & \text{if } k^* < k \le k^* + 1 \\ \Pi_{p\&t} & \text{if } k \le k^*, \end{cases}$$

with

$$V_1(k) = \frac{rBm_2^k + (1+r)a\lambda^k + (1+r)b\zeta(1-\lambda)^k}{r(\lambda^k + \zeta(1-\lambda)^k)},$$
(6)

$$V_2(k) = \frac{Bm_2^{k+1}}{(r+1)(\lambda^k + \zeta(1-\lambda)^k)} + \frac{(r+\lambda)a\lambda^k + (r+1-\lambda)b\zeta(1-\lambda)^k}{r(\lambda^k + \zeta(1-\lambda)^k)}$$

$$+\frac{\lambda(1-\lambda)(\lambda^{k-1}+\zeta(1-\lambda)^{k-1})}{\lambda^k+\zeta(1-\lambda)^k}\frac{\Pi_{p\&t}}{1+r},\tag{7}$$

and,

$$a = U_{Biorem}^{H} - C_{Biorem}; b = U_{Biorem}^{L} - C_{Biorem}, \Pi_{p\&t} = \sum_{t=0}^{\infty} \frac{U_{p\&t} - C_{p\&t}}{(1+r)^{t}} - I_{p\&t}.$$
 (8)

To determine k^* we solve the continuity condition $\lim_{k\to k^*+1}V_1(k)=V_2(k^*+1)$ and the value-matching condition $\lim_{k\to k^*}V_2(k)=\Pi_{p\&t}$. The threshold number of h-signals relative to l-signals is then given by

$$k^* = \frac{\ln\left(\frac{p^*}{1-p^*}\right) + \ln(\zeta)}{\ln\left(\frac{\lambda}{1-\lambda}\right)}.$$
(9)

A derivation of the equations is shown in the Appendix.

Results

Let us first consider the situation where the firm has already invested in the bioremediation strategy. Groundwater quality is monitored four times a year, so the net cash flow of bioremediation and the discount rate are determined on a quarterly basis. It is also assumed that the decision maker has doubts about a highly efficient bioremediation project: for the base case scenario, p_0 is set at 0.25. The probability that the samples reflect the true state of the remediation process (λ) is set at 0.7. To determine the optimal timing at which the decision maker should adopt pump & treat, the critical level k^* is calculated. In case the firm has to redirect the remediation strategy, it is assumed that the firm can recover most of the equipment installed. To adopt pump & treat, an investment cost of £15 000, instead of £142 084 (Eq. 5) is required, which results in a NPV of £151 582. Applying Equation (9), we find a k^* being equal to -2.82. Figure 4 presents the value function as a function of k graphically. It is demonstrated that since $k^* = -2.82$, it is optimal to redirect the remediation strategy and adopt the pump & treat strategy as soon as the number of low samples is 3 higher than the number of high samples. The value functions V_1 and V_2 intersect at k^*+1 . $p(k^*)$

equals 0.03 which means that as long as the belief in a high efficient bioremediation strategy is higher than 0.03, one should continue bioremediation.

Next, we analyze whether the firm would actually make the decision to initially adopt bioremediation. The value of bioremediation at the initial point in time is determined by the difference between V_1 , *i.e.* the value of bioremediation after investment (see Eq. 6), and the investment cost:

$$\frac{rBm_2^k + (1+r)a\lambda^k + (1+r)b\zeta(1-\lambda)^k}{r(\lambda^k + \zeta(1-\lambda)^k)} - I_{biorem} = \epsilon 60 \ 449$$
 (10)

with
$$a=\epsilon 4$$
 207; $b=\epsilon 382$; $\lambda=0.7$; $k=0$; $r=0.0123$; $I_{biorem}=\epsilon 128$ 667; $\zeta=\frac{1-p_0}{p_0}$; $B=314$ 847, $m_2=0.2913$.

Before the firm has installed bioremediation, no samples are taken yet, so k equals 0. In Figure 2, for k equal to 0, the value function V_1 intersects the vertical axis at \in 189 116. The difference between \in 189 116 and the bioremediation investment cost is \in 60 449. This result shows that if the option to abandon the bioremediation strategy is included in the decision analysis, the value of the initial investment decision is larger for the bioremediation strategy than for the pump & treat system (\in 24 498, Eq. 5). Hence, the firm will initially invest in the bioremediation strategy if the option to abandon is taken into account. Figure 3 presents the different results for the initial investment decision, considering a bioremediation strategy with and without the option to abandon.

4. ROBUSTNESS OF THE RESULTS

Because several parameters related to the remediation process can differ for different locations and because p_0 and λ are unknown, also for this location, parameter values are varied in a sensitivity analysis. Figure 6a shows that the larger the probability that the signal is

correct, the lower the critical belief in a good remediation project. If λ is higher, then the information provided by the samples taken is more correct. Because these samples are more informative, the firm will demand a higher certainty that the bioremediation strategy is bad, before it switches to pump & treat. Figure 6b shows that, provided λ is large enough, this belief is reached after fewer bad samples in excess of good samples are taken. In other words, the higher the probability that the samples taken are correct, the fewer bad samples in excess of good samples are required in order for the firm to switch to pump & treat. For λ smaller than 0.53, k* becomes positive: the information provided by the samples is too limited to justify a continuation of the bioremediation strategy. The firm will immediately adopt pump & treat.

If the samples taken do not provide any information (i.e. $\lambda = 0.5$), the firm will decide to switch to pump & treat from the moment its belief in an efficient bioremediation strategy is lower than 0.38. The value of the critical belief in this case is determined as follows (see Eq. 28 in Appendix),

$$\lim_{\lambda=0.5} p^* = \frac{r\Pi_{p\&t}}{1+r} - b = 0.38,$$
with $a = \text{ } 4207; b = \text{ } 382.$

Figure 7 shows for different values of λ and p_0 when the firm is indifferent between continuing bioremediation or switching to pump & treat (k* = 0). For the area below the curve, k* is positive: both the correctness of the samples taken and the prior belief in a good bioremediation strategy are too low to justify a continuation of the bioremediation strategy. Hence, if the prior belief in an efficient bioremediation strategy is low, it is important that the samples taken are correct. Otherwise, one will immediately adopt pump & treat. If the prior

belief in a good bioremediations strategy is larger than 0.38, the initial decision of the firm will be to invest in bioremediation, even if the samples taken do not provide any information.

Figure 8 presents V_1 as a function of k, separated into the value of the option to adopt pump & treat while using bioremediation and the value of the bioremediation strategy. The more bioremediation proves to be inefficient, the more valuable it is to have the option to redirect the remediation strategy. The option value is very low when the number of good samples in excess of bad samples is higher than 6.

To determine the annual incoming cash flows listed in Table 2, the valuation of groundwater remediation is based on a property value of €23 m⁻². This value can vary geographically or can be different for different types of land use. Therefore, the effect of variations of this parameter value on k* is examined. Figure 9a makes clear that if the value is smaller than €18 m⁻², it will never be optimal to abandon bioremediation and switch to pump & treat, because the NPV of pump & treat is always smaller than the value of bioremediation, regardless of the bioremediation efficiency. If this value is higher than €43 m⁻², the decision maker will always adopt the pump & treat strategy, because the corresponding NPV is always larger than the one of the bioremediation strategy. The reason is that when revenues after clean-up are large, the larger investment and operational costs for pump & treat are justificied. Note that pump & treat removes a larger annual quantity of contaminated mass than bioremediation, even when bioremediation is regarded to be efficient. For a property value of €36 m⁻², the firm is indifferent between continuing bioremediation and adopting pump & treat. Regarding the initial investment decision, Figure 9b shows that without option to abandon, the firm would only decide to adopt bioremediation if the property value is smaller than €21 m⁻². With option to abandon, bioremediation is the preferred remediation strategy for property values smaller than €43 m⁻². Note that the difference in NPV between P&T and bioremediation becomes very small for increasing property values. In Flanders agricultural land is sold at an average price of €2.8 m⁻² and land values in industrial areas range between €25 m⁻² and €70 m⁻². Hence, for contaminated parcels in these areas, bioremediation might be a viable remediation strategy. In order for bioremediation to be selected as a groundwater remediation strategy in these areas, the Flemish authorities should allow for incorporating flexibility when the remediation project is drafted.

For the base case scenario, an investment cost of €15 000 is assumed when the firm decides to switch from bioremediation to pump & treat. Figure 10a shows that once the investment cost is larger than €135 000, the firm would never consider to switch to pump & treat, because the NPV of pump & treat is always smaller than the value of the bioremediation strategy. Regarding the initial investment decision, Figure 10b shows that without the option to abandon, the firm would never decide to adopt the bioremediation strategy. When the option to abandon is included, the firm will adopt bioremediation only if the investment cost to redirect the remediation strategy and switch to pump & treat is smaller than €70 000.

For the base case scenario, the efficiency of the bioremediation project varies between 2.7 and 4.0 ton of contaminant mass annually removed. Figure 11a shows that if the upper limit would drop to less than 3.2 ton, the decision maker should immediately switch to pump & treat because in that case, the NPV of pump & treat will always be larger than the value of the bioremediation strategy. Regarding the initial investment decision, Figure 11b shows that when the maximum attainable efficiency level is smaller than 4.7 ton contaminated mass annually removed, it is necessary to integrate the option to abandon the bioremediation strategy when developing the remediation project.

Concerning the initial investment decision, p_0 is set to 0.25 for the base case scenario. Figure 12 demonstrates how the initial investment decision varies for different values of p_0 . If p_0 is larger than 0.39, the decision maker invests in the bioremediation strategy, even if the option to abandon does not exist. If the initial belief in an efficient Bioremediation strategy varies between 0.05 and 0.39, the option to redirect the remediation strategy is required for the decision maker to invest in the bioremediation strategy.

DISCUSSION

This study considers the investment in an innovative project under technological uncertainty. Technological uncertainty is gradually resolved in a Bayesian process by evaluating the project at determined points in time. Only when the firm has the possibility to reconsider the investment in an innovative technique, it is optimal to adopt the new technology. During the time that the innovative technology operates, one can observe its efficiency and hence learn from its application.

For the case study considered, it is assumed that contaminant mass removal continues indefinitely. Also the quantity of contaminant mass already removed by the bioremediation strategy before the decision to redirect the remediation strategy is not taken into account. However, this aspect is case specific. Note that there exist other environmental friendly technologies that operate continuously (e.g. emission abatement technologies) for which this real options model can be applied as well.

To get more insight into how the integration of the option to abandon stimulates the adoption and diffusion of new technologies, further research is required. Nevertheless, this study

demonstrates how the integration of the real options theory in the investment decision can evaluate the development of innovative clean technologies. This analysis should be considered as a tool to spur the escape from technological lock-in and to keep different technological options open.

This tool not only supports the decision making process within firms, it can also be used by governmental authorities that aim to increase investments in environmental friendly remediation technologies. By allowing firm to incorporate flexibility when developing the remediation project, experimentation with alternative technologies is encouraged, which can lead to technological change.

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APPENDIX

Given that even after an 1-sample arrives it is not optimal to abandon the bioremediation strategy, *i.e.*, k > k*+1. Because the time steps are discrete, the value function of the opportunity to adopt pump & treat for the firm, denoted by V_1 , must satisfy the following Bellman equation:

$$V_1(k) = \frac{1}{1+r} E(V_1(k')|k) + \Pi(k), \quad k > k^* + 1,$$
(11)

with k' being the quantity of good samples in excess of bad samples in the next period.

We have that

$$\Pi(p(k)) = p(k)a + (1 - p(k))b, \tag{12}$$

$$a = U_{Biostim}^H - C_{Biostim}$$
; $b = U_{Biostim}^L - C_{Biostim}$,

and

$$E(V_1(k')|k) = p(k) (\lambda V_1(k+1) + (1-\lambda)V_1(k-1))$$

$$+ (1-p(k)) (\lambda V_1(k-1) + (1-\lambda)V_1(k+1)).$$
(13)

Substitution of Eq. (6) and Eq. (7) into (5) and rewriting gives

$$(1+r)V_1 = (1+r)(p(k)a + (1-p(k))b) + (2p(k) + 1 - \lambda - p(k))V_1(k+1)$$
$$+ (p(k) + \lambda - 2\lambda p(k))V_1(k-1).$$
(14)

By substituting Eq. (3) the term associated with $V_1(k+1)$ can be written as

$$2p(k)\lambda + 1 - \lambda - p(k) = \frac{\lambda^{k+1} + \zeta(1-\lambda)^{k+1}}{\lambda^k + \zeta(1-\lambda)^k}.$$
 (15)

Similarly, we can rewrite the term associated with $V_1(k-1)$ as

$$p(k) + \lambda - 2p(k)\lambda = \frac{\lambda(1-\lambda)(\lambda^{k-1} + \zeta(1-\lambda)^{k-1})}{\lambda^k + \zeta(1-\lambda)^k}.$$
 (16)

Substituting Eq. (15) and Eq. (16) in Eq. (14) and defining $F(k) = (\lambda^k + \zeta(1-\lambda)^k)V_1(k)$, yields

$$(r+1)F(k) = (1+r)(a\lambda^k + b\zeta(1-\lambda)^k) + F(k+1) + \lambda(1-\lambda)F(k-1).$$
 (17)

The solution of the homogenous equation is given by

$$F(k) = Am_1^k + Bm_2^k, \ m_1 \neq m_2, \tag{18}$$

where A and B are constants and m₁ and m₂ are the roots of the homogenous equation

$$Q(m) \equiv -(r+1)m + m^2 + \lambda(1-\lambda) = 0. \tag{19}$$

Eq. 13 has two roots, namely

$$m_{1,2} = \frac{(r+1)}{2} \pm \frac{\sqrt{(r+1)^2 - 4\lambda(1-\lambda)}}{2}.$$
 (20)

Note that Q(m) is an upward pointing parabola (a>0) with $Q(0) = \lambda(1-\lambda) > 0$ and $Q(\lambda) = -r\lambda \le 0$. Thus it holds that $m_1 > \lambda$ and $m_2 < \lambda$.

When the number of h-signals relative to l-signals tends to infinity, the value of the option to invest in pump & treat converges to zero, i.e.

$$\lim_{k \to +\infty} \frac{Am_1^k + Bm_2^k}{\lambda^k + \zeta(1 - \lambda)^k} = 0.$$
 (21)

This implies that we only need to consider the smaller root m_2 , so that A=0.

The particular solution of Eq. (11) yields

$$F(k) = \frac{a\lambda^k (1+r)}{r} + \frac{b\zeta(1-\lambda)^k (1+r)}{r}.$$
(22)

The value function V_1 is then given by

$$V_1(k) = \frac{rBm_2^k + (1+r)a\lambda^k + (1+r)b\zeta(1-\lambda)^k}{r(\lambda^k + \zeta(1-\lambda)^k)}.$$
 (23)

Given that $k^* < k \le k^* + 1$. The value of k is such that it is not optimal to invest in pump & treat right away. However, when the following sample is an l-sample, it will be optimal to abandon bioremediation. Analogous to Eq. (11) the Bellman equation for $V_2(k)$ is given by

$$V_2(k) = \frac{1}{1+r} E(V_2(k')|k) + \Pi(k), \quad k > k^* + 1.$$
(24)

We have that

$$E(V_{2}(k')|k) = p(k)(\lambda V_{1}(k+1) + (1-\lambda)\Pi_{p\&t}) + (1-p(k))(\lambda \Pi_{p\&t} + (1-\lambda)V_{1}(k+1)),$$
(25)

so that

$$V_{2}(k) = \frac{Bm_{2}^{k+1}}{(r+1)(\lambda^{k} + \zeta(1-\lambda)^{k})} + \frac{(r+\lambda)a\lambda^{k} + (r+1-\lambda)b\zeta(1-\lambda)^{k}}{r(\lambda^{k} + \zeta(1-\lambda)^{k})} + \frac{\lambda(1-\lambda)(\lambda^{k-1} + \zeta(1-\lambda)^{k-1})}{\lambda^{k} + \zeta(1-\lambda)^{k}} \frac{\Pi_{p\&t}}{1+r}.$$
(26)

If an 1-sample arrives, the process jumps to the region where $k \le k^*$ and if an h-sample arrives, the process jumps to the region where $k > k^* + 1$. Therefore, the value V_2 is completely determined by $V_1(k+1)$ and $\prod_{p\&t}$.

The value function V then equals

$$V(k) = \begin{cases} V_1(k) & \text{if } k > k^* + 1 \\ V_2(k) & \text{if } k^* < k \le k^* + 1 \\ \Pi_{p\&t} & \text{if } k \le k^*, \end{cases}$$

where $V_1(k)$, $V_2(k)$ and $\prod_{p\&t}$ are given by equations (4), (17), and (20) respectively. To determine B and k^* we solve the continuity condition $\lim_{k\to k^*+1} V_1(k) = V_2(k^*+1)$ and the value-matching condition $\lim_{k\to k^*} V_2(k) = \prod_{p\&t}$. The latter equation yields

$$B = \frac{\prod_{p \& t} r \left[\lambda^{k^*} (\lambda + r) - \zeta (1 - \lambda)^{k^*} (\lambda - r - 1) \right]}{r m^{k^* + 1}} - \frac{a \lambda^{k^*} (r + \lambda) (1 + r) - b \zeta (1 - \lambda)^{k^*} (\lambda - r - 1) (1 + r)}{r m^{k^* + 1}}.$$
 (27)

Substituting B in the former equation leads to an expression for $p^* \equiv p(k^*)$:

$$p(k^*) = \frac{X}{Y},\tag{28}$$

where

$$\begin{split} X &= \left(\mathrm{br} + \mathrm{b} - \Pi_{\mathrm{p\&t}} r \right) [(\lambda - 1)^2 + m_2 (\lambda - r - 1) + r^2 - (\lambda - 2) r], \\ Y &= m_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(-\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r + 1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)(\lambda + r) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1) + (1-2\lambda) \Pi_{\mathrm{p\&t}} r \right) + n_2 \left(a(r+1)(\lambda + r) - b(r+1)$$

$$(r+1)\left(-a(\lambda^2+r^2+\lambda r+r)+b((\lambda-1)^2+r^2-(\lambda-2)r)+(2\lambda-1)\Pi_{p\&t}r\right).$$

The threshold number of h-signals relative to l-signals is then given by

$$k^* = \frac{\ln\left(\frac{p^*}{1-p^*}\right) + \ln(\zeta)}{\ln\left(\frac{\lambda}{1-\lambda}\right)}.$$
 (29)