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A Hotelling model for the circular economy including recycling, substitution and waste accumulation

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Abstract

Non-renewable resources include a large variety of deposits that have been formed by geological processes over millions of years. Although extraction of such resources provides benefits as employment and economic revenues, it also contributes to negative environmental externalities and it increases resource scarcity. An important policy question is how to optimally extract non-renewable resource stocks over time while taking possible substitutes and recycling into account. The present paper adds to the literature by developing a generic numerical optimisation model that can be used to simulate non-renewable resource management regimes and the effects of different policy instruments deployed at different stages of the resource's life cycle. By including recycling and substitution, the model extends the seminal cake-eating Hotelling model that dominates the non-renewable resource economics literature. In addition to being generically designed, the model can accommodate for non-competitive market settings, interacting policy instruments and environmental externalities at different stages of the material's life cycle. The model's possibilities are illustrated by means of a numerical simulation example for the extraction of sand.

Keywords

Non-renewable resources, Hotelling, Recycling, Substitution, Dynamic Numerical Optimisation Modelling

1 Introduction

Non-renewable resources include a large variety of mineral deposits from which metals, fossil fuels and other processed minerals can be obtained. Although the extraction of these resources provides local employment and revenues, it is usually accompanied by negative environmental externalities. For example, quarrying sand and gravel can be noisy and dusty and traffic to the mining pit can create disamenities for neighbours. Furthermore, the natural environment can be damaged by biodiversity loss, run-off water, waste generation and visual pollution (Eckermann et al., 2012). Along with these negative aspects is often a problem of scarcity. As the crude forms of these non-renewable resources were created by long-term geological processes, their rate of formation is so slow – in timescales relevant to humans – that they should be labelled as non-renewable (Perman et al., 2011). In addition, the intensive use of these resources that formed the basis of economic prosperity in many developed countries, and strict demarcations of mining areas, causes remaining reserves to be limited and scarce (European Commission, 2011a). The European Union has recognised that the current rate of extraction of non-renewable resources is not sustainable and it has identified resource efficiency as one of seven flagship projects to pursue in its Europe 2020 strategy (European Commission, 2011b). This flagship initiative, which has the aim of creating frameworks for policies to support the shift towards a more resource-efficient and low-carbon economy, raises the key policy question: what is the optimal extraction path over time of a non-renewable resource in a circular economy[1] setting?

There is no straightforward answer to this question because non-renewable resources are heterogeneous and it is often unclear what policies should be undertaken in order to facilitate the transition towards a resource-efficient economy. The prevailing view is that increasing

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[1] See for example Ellen MacArthur Foundation (2015), Stahel (2016) or Van Acker et al. (2016) for attempts to define the concepts of circular economy and resource efficiency in more detail.

scarcity of non-renewable resources will be accompanied by a steady price increase that signals scarcity to consumers and provides incentives for eco-innovations for substituting or limiting the use of scarce materials. However, the incentives given by the price mechanism are often fundamentally flawed when it comes to the reaction of private sectors. Private resource owners are often more impatient than society as a whole, which leads to excessively fast exploitation. In addition, market prices often reflect insufficiently environmental externality costs in the absence of proper government regulation (Eyckmans and Dubois, 2014, Söderholm and Tilton, 2012). Based on these observations, implementing policy instruments to foster more sustainable resource use is justified. Moreover, this is in accordance with the calls for ‘true pricing’ by internalising external costs and with the green tax shift debate. At present, many European Member States have not made a substantial shift from labour towards environmental taxation, even though environmental taxes can be a step towards reflecting the full external and social costs of resource extraction, utilisation and end-of-life practices (Bringezu, 2002; Wilts et al., 2014). Along with steering behaviour, these taxes would help to reorientate public finances away from labour taxation, which could benefit job creation and economic growth.

The discussion so far highlights the difficulty of identifying policies that trigger the transition towards a resource-efficient, circular economy. The challenge is exacerbated by the lack of appropriate methodologies that combine phenomena such as resource extraction, environmental externalities, waste accumulation, recycling and substitution in a unified framework. This paper intends to add to the existing literature by developing a generic optimisation model that can be used to simulate non-renewable resource regimes and the effects that different policy instruments can have within the material flow of a particular substance. The generic optimisation model provides a tool for designing policies that foster the transition towards a more resource-efficient economy, which can boost economic performance while reducing resource use and negative environmental externalities.

Section two describes in detail the modelling framework. In the third section, numerical simulations are presented, illustrating the capabilities of the modelling framework. A discussion of the model's capabilities and limitations and of interesting future research topics is presented in section four. Section five concludes the article with an overview of the most important findings.

2 Hotelling model with recycling

Numerical models often serve as a bridge between theoretical models and analyses of real-world policy questions. In addition, numerical optimisation problems are often used to quantify the net effects of counteracting forces that theoretical models are unable to sign unambiguously (Conrad, 1999; Epple and Londregan, 1993; Flakowski, 2004). Although such optimisation problems are actually simplified representations of reality, they can provide generally applicable and policy-relevant insights into how to foster resource efficiency by implementing an appropriate mix of policy instruments. The basis of the model developed in this chapter lies with the well-known Hotelling model (Hotelling, 1931). According to the Hotelling rule, the shadow price of a non-renewable resource should increase at the rate of discount along the socially optimal extraction path. This rising shadow price reflects the increasing opportunity cost as remaining non-renewable resource reserves are consumed. Private profit maximising resource owners interacting on a competitive commodity market will choose an extraction path that coincides with the socially optimal one provided the private and social discount rates are equal (Chermak and Patrick, 2002; Perloff, 2011).

Already in the 1970s, several theoretical models on resource extraction and recycling were developed. In a study carried out by Smith (1972) for example, a rudimentary model was used that emphasises only those elements essential to the recycling problem. Later, Lusky (1975) developed an integrated model of conservation and recycling in a framework of a natural resource cycle, and Hoel (1978) studied the optimal path of extraction and recycling under various assumptions about the environmental effects of recycling and the assimilative capacity of the environment. In addition to these theoretical models, also numerical simulation models in the same spirit were published. In the study by Weikard and Seyhan (2009) for example, a resource extraction model was built for a competitive fertilizer market including different recycling options. Seyhan et al. (2012) also focused on the extraction and recycling of Phosphorus, and developed a resource-specific model. Compared to these studies, our model develops a comprehensive generic optimisation model that can be used to simulate non-renewable resource regimes and effects of different policy instruments within the material flow of a particular resource. Our model includes recycling, substitution and waste accumulation in a unified framework, and is able to simulate different scenarios like non-competitive market settings, first-best welfare maximisation scenarios, interacting policy instruments and environmental externalities linked to different stages of the material flow.

2.1 Economic actors in decentralised market model

The model involves four different types of economic actors: (i) consumers, (ii) resource owners, (iii) suppliers of substitute material and (iv) recyclers.

2.1.1 Consumers

We assume a large number of identical consumers. The representative consumer chooses to consume an amount of non-renewable resources, $\boldsymbol{Q_t}$, to maximise its utility while taking into account its budget constraints. In the model, preferences for consumption are represented by an increasing and strictly concave utility function $\boldsymbol{U(Q_t)}$, so that $\boldsymbol{U' \geq 0}$ and $\boldsymbol{U'' < 0}$. Furthermore, there is a numéraire good, $\boldsymbol{v_t}$, the price of which is normalised to unity. Making use of this numéraire good facilitates comparisons as all relative prices in the model can be expressed in terms of this numéraire as a tradable economic commodity. It is further assumed that the income of the consumers is exogenous and that no intertemporal savings or borrowing take place. In the model, the exogenous income is denoted by $\bar{\boldsymbol{y}}_{\boldsymbol{t}}$ and is strictly larger than zero. The price of the good is denoted by $\boldsymbol{p}_{\boldsymbol{t}}$, and can be supplemented with a consumption excise tax $\boldsymbol{t}_{\boldsymbol{t}}^{\boldsymbol{q}}$. We assume there is a waste market where recycling companies try to acquire discarded consumption products for recycling the embedded material. In order to introduce this waste market we foresee the possibility that consumers are paid a price $\boldsymbol{p}_{\boldsymbol{t}}^{\boldsymbol{w}}$ for their end of life consumption products $\boldsymbol{w}_{\boldsymbol{t}}$. Note however that in the waste market equilibrium, this waste price can be negative meaning that the consumer would be charged a price for disposing waste instead of receiving money for handing over end of life products to the recyclers. In the section on recyclers we will discuss in detail the determinants of this equilibrium waste price. Combining all these elements provides the following constrained utility optimisation problem in period $\boldsymbol{t}$:

$$\boldsymbol{max}_{\boldsymbol{v}_{\boldsymbol{t}},\boldsymbol{Q}_{\boldsymbol{t}}}\ \boldsymbol{v}_{\boldsymbol{t}}+\boldsymbol{U}(\boldsymbol{Q}_{\boldsymbol{t}}) \quad \boldsymbol{s.t.}\ \ \boldsymbol{v}_{\boldsymbol{t}}+\left[\boldsymbol{p}_{\boldsymbol{t}}+\boldsymbol{t}_{\boldsymbol{t}}^{\boldsymbol{q}}\right]\boldsymbol{Q}_{\boldsymbol{t}}-\boldsymbol{p}_{\boldsymbol{t}}^{\boldsymbol{w}}\boldsymbol{w}_{\boldsymbol{t}}\leq\bar{\boldsymbol{y}}_{\boldsymbol{t}} \tag{1}$$

Assuming that consumption goods only lasts for one period[2], we can replace $\boldsymbol{w}_{\boldsymbol{t}}$ by $\boldsymbol{Q}_{\boldsymbol{t}}$ and the corresponding Lagrangian function of this consumer problem is given by:

$$\boldsymbol{L}(\boldsymbol{v}_{\boldsymbol{t}},\boldsymbol{Q}_{\boldsymbol{t}},\boldsymbol{\lambda}_{\boldsymbol{t}})=\boldsymbol{v}_{\boldsymbol{t}}+\boldsymbol{U}(\boldsymbol{Q}_{\boldsymbol{t}})+\boldsymbol{\lambda}_{\boldsymbol{t}}[\,\bar{\boldsymbol{y}}_{\boldsymbol{t}}-\boldsymbol{v}_{\boldsymbol{t}}-[\boldsymbol{p}_{\boldsymbol{t}}+\boldsymbol{t}_{\boldsymbol{t}}^{\boldsymbol{q}}-\boldsymbol{p}_{\boldsymbol{t}}^{\boldsymbol{w}}]\boldsymbol{Q}_{\boldsymbol{t}}] \tag{2}$$

In equation (2), parameter $\boldsymbol{\lambda}_{\boldsymbol{t}}$ represents the Lagrange multiplier of the consumer's budget constraint or marginal utility of extra income. Taking the derivative of the Lagrangian with respect to the numéraire good $\boldsymbol{v}_{\boldsymbol{t}}$, it follows directly that $\boldsymbol{\lambda}_{\boldsymbol{t}}=\boldsymbol{1}$. The relevant Karush-Kuhn-Tucker first-order conditions for a utility maximum, taking into account the non-negativity constraint in consumption $\boldsymbol{Q}_{\boldsymbol{t}}$, can be written as:

$$\boldsymbol{U}'(\boldsymbol{Q}_{\boldsymbol{t}})-\boldsymbol{p}_{\boldsymbol{t}}-\boldsymbol{t}_{\boldsymbol{t}}^{\boldsymbol{q}}+\boldsymbol{p}_{\boldsymbol{t}}^{\boldsymbol{w}}\leq\boldsymbol{0},\quad \boldsymbol{Q}_{\boldsymbol{t}}\geq\boldsymbol{0},\ \left[\boldsymbol{U}'(\boldsymbol{Q}_{\boldsymbol{t}})-\boldsymbol{p}_{\boldsymbol{t}}-\boldsymbol{t}_{\boldsymbol{t}}^{\boldsymbol{q}}+\boldsymbol{p}_{\boldsymbol{t}}^{\boldsymbol{w}}\right]\boldsymbol{Q}_{\boldsymbol{t}}=\boldsymbol{0} \tag{3}$$

Basically, equation (3) says that in case of an interior solution $\boldsymbol{Q}_{\boldsymbol{t}}>\boldsymbol{0}$, consumers will buy consumption goods up to the point at which their marginal utility of consumption equals the full consumer price of the good. This consumer price consists of the purchasing price $\boldsymbol{p}_{\boldsymbol{t}}$,

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[2] More sophisticated ways of modelling the intertemporal link between consumption and ensuing waste are discussed in section 2.2.

supplemented with the consumption excise tax $\boldsymbol{t_t^q}$, minus (plus) the waste price (charge) $\boldsymbol{p_t^w}$. In case $\boldsymbol{U'(0) < p_t + t_t^q - p_t^w}$, consumers will not buy as the price exceeds their maximum marginal willingness to pay. This formulation is very practical for functional forms of utility and demand functions that imply a choke-off price. When this choke-off price is reached, the quantity demanded falls to zero, meaning that demand is choked off at this price. The intuition for such a choke-off price is that people switch to a substitute consumption good if the market price exceeds the choke-off price. With regard to the numerical implementation of the demand for the consumption good, the standard implementation of the model uses a linear inverse demand curve but other functional forms can easily be implemented as well. Using the linear formulation for the demand function allows a straightforward interpretation of the choke-off price as the intercept of the inverse demand function with the price axis. Note that we we allow for the possibility that the intercept and the slope of the demand curve change over time; for instance, in order to reflect changes in real income, preferences or population over time. This gives following demand function:

$$\boldsymbol{U'(Q_t) = a_t - b_t Q_t} \qquad (4)$$

The utility function necessary to calculate welfare and corresponding with this inverse demand function is given by the integral under the marginal utility function:

$$\boldsymbol{U(Q_t) = \int_0^{Q_t} U'(x)dx = \int_0^{Q_t} [a_t - b_t x]dx = a_t Q_t - \frac{b_t}{2} Q_t^2 + \mathbf{constant}} \qquad (5)$$

Assuming an interior solution $\boldsymbol{Q_t > 0}$ and differentiating of the first-order equation (3) shows that, ceteris paribus, the utility maximizing consumption level $\boldsymbol{Q_t}$ decreases when the price $\boldsymbol{p_t}$ or excise tax rate $\boldsymbol{t_t^q}$ increases, and that it increases when the price of waste increases:

$$\boldsymbol{U'' dQ_t = dp_t + dt_t^q - dp_t^w \quad \Rightarrow \quad \frac{dQ_t}{dt_t^q} = \frac{dQ_t}{dp_t} = \frac{1}{U''} < 0 \quad \mathbf{and} \quad \frac{dQ_t}{dp_t^w} = \frac{-1}{U''} > 0} \qquad (6)$$

2.1.2 Mining companies

A second type of economic actors are the resource owners. They extract the non-renewable resource as virgin material and sell it directly to the consumers. The quantity of virgin extraction by a representative resource owner is denoted by $\boldsymbol{q_t^v}$. The total initial stock of this virgin material is given by $\boldsymbol{S_0}$ and is assumed strictly positive. As this total stock is fixed, the model can be classified as a kind of cake-eating model of non-renewable resource depletion (Weikard and Seyhan, 2009). In each period, mining companies decrease the remaining stock by extracting virgin resources. At every moment in time, this remaining stock should be nonnegative. Using a linear demand function, if follows that virgin resource extraction will stop in finite time at period $\boldsymbol{t = T}$ (see Conrad 1999). The marginal cost of virgin material production is assumed to be constant, i.e. independent of the quantity produced, at every point in time. We foresee however the possibility that the marginal production costs decreases over time as a result of technological progress[3]. In the model, this marginal production cost is represented by parameter $\boldsymbol{c_t^v}$. Next to this cost parameter, we foresee the possibility of introducing a virgin material extraction tax $\boldsymbol{t_t^v}$. The related environmental motives for taxing resource extraction identified in the literature are: (i) to decrease the rate of extraction, (ii) to focus on all generated environmental externalities and (iii) to encourage the substitution of secondary and recycled materials for virgin material (Söderholm, 2011).

The mining sector itself is modelled as a standard Hotelling non-renewable resource problem, with every mining company maximising its sum of future discounted profits. With $\boldsymbol{\delta_t = \frac{1}{[1+\rho]^t}}$ denoting the private discount factor and $\boldsymbol{\rho}$ the private discount rate, mine owners decide when to extract and sell the mined, non-renewable resources in order to maximise the present value of the resource. This gives rise to the following maximisation problem:

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[3] More sophisticated cost functions are easy to implement in the numerical model like costs that increase in the cumulative extraction of the non-renewable resource, see for example Conrad (1999).

$$max_{q_t^v\,(t=1,2,\dots,T)}\ \pi^v = \sum_{t=1}^{T} \delta_t[p_t - c_t^v - t_t^v]q_t^v \tag{7}$$

$$\text{s.t.}\quad \begin{cases} S_{t+1} - S_t = -q_t^v \quad \forall t = \{1,2,\dots,T\},\quad S_0 > 0 \\ S_t \geq 0 \quad \forall \mathrm{t} = \{1,2,\dots,T\} \\ q_t^v \geq 0 \quad \forall \mathrm{t} = \{1,2,\dots,T\} \end{cases}$$

The first restriction in maximization problem (7) is the equation of motion of the resource stock. It states that the remaining resource stock at the beginning of period t+1 is equal to the remaining stock at the beginning of previous period t, minus the virgin extraction that takes place in period t. The second restriction ensures that the total supply of virgin material over time does not exceed the initially available quantity S_0. Writing the Lagrangian for this dynamic program gives us:

$$L = \pi^v = \sum_{t=1}^{T} \delta_t[p_t - c_t^v - t_t^v]q_t^v - \sum_{t=1}^{T} \delta_{t+1}\lambda_{t+1}[S_{t+1} - S_t + q_t^v] \tag{8}$$

In equation (8), the Lagrange multiplier of the resource stock's equation of motion was, without loss of generality, multiplied by the discount factor δ_{t+1} in order to simplify calculations. Taking into account the non-negativity constraints for the virgin material extraction rate (control variable) and the remaining resource stock (state variable), the relevant Karush-Kuhn-Tucker first-order conditions can be written as follows:

$$\frac{\partial L}{\partial q_t^v} = \delta_t[p_t - c_t^v - t_t^v] - \delta_{t+1}\lambda_{t+1} \leq 0,\quad q_t^v \geq 0,\quad [\delta_t[p_t - c_t^v - t_t^v] - \delta_{t+1}\lambda_{t+1}]q_t^v = 0 \tag{9}$$

$$\frac{\partial L}{\partial S_t} = \delta_{t+1}\lambda_{t+1} - \delta_t\lambda_t \leq 0,\quad S_t \geq 0,\quad [\delta_{t+1}\lambda_{t+1} - \delta_t\lambda_t]S_t = 0 \tag{10}$$

The first-order condition with respect to the state variable S_t can be rewritten as:

$$\lambda_{t+1} - \lambda_t - \lambda_t\rho \leq 0,\quad S_t \geq 0,\quad [\lambda_{t+1} - \lambda_t - \rho\lambda_t]S_t = 0 \tag{11}$$

Similarly, the first-order condition with respect to the control variable q_t^v can be rearranged:

$$p_t - c_t^v - t_t^v - \lambda_t \leq 0,\quad q_t^v \geq 0,\quad [p_t - c_t^v - t_t^v - \lambda_t]q_t^v = 0 \tag{12}$$

In these equations, parameter λ_t represents the shadow price of the resource. Assuming an interior solution, the latter two equations can be combined yielding the well-known Hotelling rule for the optimal extraction of a non-renewable resource:

$$\frac{\lambda_{t+1}-\lambda_t}{\lambda_t} = \rho \quad \Leftrightarrow \quad \frac{[p_{t+1}-c_{t+1}^v-t_{t+1}^v]-[p_t-c_t^v-t_t^v]}{[p_t-c_t^v-t_t^v]} = \rho \tag{13}$$

Equation (13) shows that, along an optimal extraction path, the shadow price of the non-renewable resource increases at the rate of discount ρ. In other words, the discounted net price of this non-renewable resource is constant along the efficient resource extraction path. By formulating the Hotelling rule in this way, it can be seen that the Hotelling rule is actually a special case of a general asset-efficiency condition. In particular, this condition states that the present value of any efficiently managed asset should be constant over time.

2.1.3 Substitute suppliers

A third type of economic actors are the suppliers of the substitute. This substitute material can be for example imported material from abroad. Substitution will take place when the price of the non-renewable virgin resource rises to such level that it makes alternative sources of supply economically more attractive. Would a substitute come to the market, its full price would function as a choke-off price, at which a switch is made from virgin to substitute material. The quantity of the substitute is represented by variable q_t^s. We assume that this substitute material can be imported at a fixed cost c_t^s And that its supply is perfectly elastic. Next to this cost parameter, we foresee the possibility that authorities levy an import duty t_t^s on the material. The supply schedule of the substitute material is given by the following Karush-Kuhn-Tucker first-order condition:

$$p_t - c_t^s - t_t^s \leq 0, \quad q_t^s \geq 0, \quad [p_t - c_t^s - t_t^s]q_t^s = 0 \tag{14}$$

This condition implies that if the substitute material comes onto the market $\boldsymbol{q_t^s > 0}$, it holds that $\boldsymbol{p_t = c_t^s + t_t^s}$. Otherwise, if the price is lower than the sum of import costs and duties $\boldsymbol{p_t < c_t^s + t_t^s}$, the substitute material will not come to the market and $\boldsymbol{q_t^s = 0}$.

2.1.4 Recyclers

Apart from virgin and substitute material, we also consider recyclers that process end-of-life waste with the intention of producing recycled material that can compete with virgin material. We assume that there is a market for waste, i.e. discarded end-of-life consumer goods, where the recycler can source waste from consumers for processing in its recycling facility. Furthermore, we assume that there is no free disposal of waste in terms of illegal dumping or street litter and that there is no retention of waste with consumers[4]. In processing the waste, represented by variable $\boldsymbol{w_t}$, a representative recycler chooses its recycling effort $\boldsymbol{\beta_t}$ as to maximise profits. As $\boldsymbol{\beta_t}$ represents the share of material that is extracted from the waste, its value lies in the range [0,1]. The revenue of the recyclers consists of proceeds from selling recycled material at price $\boldsymbol{p_t}$. At the same time, the recyclers bear different costs. In the model we assume that recycling has an increasing and convex cost function $\boldsymbol{r(\beta_t)}$, so that $\boldsymbol{r' \geq 0}$ and $\boldsymbol{r'' > 0}$, with $\boldsymbol{r}$ representing the recycling unit cost that is an increasing and strictly convex function of recycling effort $\boldsymbol{\beta_t}$. The non-recyclable fraction is disposed of at a price $\boldsymbol{p_t^d}$ per unit. This parameter includes the gate fee that is charged at the landfill and a possible landfill or disposal tax. Together with the extraction tax, the tax on waste disposal could provide strong incentives to employ recycled materials rather than to extract virgin materials (Ecotec, 2001; Söderholm, 2011). Finally, we allow for the possibility that recyclers are taxed (or subsidized) on

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[4] If illegal waste disposal is possible, full pass through of external costs is typically impossible and second-best levels of environmental taxation have to be considered. Illegal behaviour at the consumer side is not the focus of our paper and we refer interested reader to Fullerton and Kinnaman (1995) for a formal analysis of illegal waste disposal and recycling.

their recycling activities at rate $\boldsymbol{t_t^r}$ per unit of waste they process. Summarising, a representative recycler solves the following profit maximisation problem:

$$\boldsymbol{max_{\beta_t}\ \pi_t^r = \{p_t\beta_t w_t - p_t^w w_t - r(\beta_t)w_t - [1-\beta_t]w_t p_t^d - t_t^r w_t\}} \tag{15}$$

Taking the derivative of this equation with respect to recycling effort $\boldsymbol{\beta_t}$ gives rise to the following first-order condition:

$$\boldsymbol{p_t - r'(\beta_t) + p_t^d = 0} \tag{16}$$

In a competitive recycling market, the marginal cost of recycling, $\boldsymbol{r'(\beta_t)}$ should be equal to the price of the virgin resource plus the full cost of landfilling. Every extra percent of recycling generates an extra unit of recycled material and avoids a unit of residuals that are send to the landfill. Note that we assume here that recycled material is of equal quality as virgin material (perfect substitutes) such that they can command the same price in the material's market. In the model, it is assumed that $\boldsymbol{r(0) = 0}$, $\boldsymbol{r'(0) = 0}$ and that the limit of $\boldsymbol{r'(\beta_t)}$ tends to plus infinity when $\boldsymbol{\beta_t}$ approaches one[5]. This ensures the existence of an interior solution if $\boldsymbol{p_t + p_t^d}$ is strictly larger than zero. Totally differentiating equation (16) yields:

$$\boldsymbol{r'' d\beta_t = dp_t + dp_t^d \quad \Rightarrow \quad \frac{d\beta_t}{dp_t} = \frac{d\beta_t}{dp_t^d} = \frac{1}{r''} > 0} \tag{17}$$

This equation reveals intuitive ceteris paribus comparative statics results: the higher the price of material (assuming the waste disposal price remaining the same), the higher the recycling effort chosen by a profit-maximising recycling firm. Similarly, the higher the price of disposal of recycling residues, the higher the recycling effort chosen by the recyclers for a given material price. These increasing recycling efforts reduce the pressure on demand for virgin materials, help to reuse valuable materials that would otherwise be wasted, and reduce energy consumption and greenhouse gas emissions from extraction and processing (European

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[5] For the recycling unit cost function we use as functional form $\boldsymbol{r(\beta_t) = [-g_t]\big[[1-\beta_t]log(1-\beta_t) + \beta_t\big]}$ with parameter $\boldsymbol{g_t < 0}$. The resulting marginal cost function is given by $\boldsymbol{r'(\beta_t) = g_t log(1-\beta_t)}$. This functional form satisfies all the limit conditions assumed in the theoretical model.

Commission, 2011; Pittel et al., 2010). Note that the waste price p_t^w and tax (or subsidy) on recycling activities t_t^r does not impact the recycling effort β_t because the recycler pays the consumer and the recycling tax per unit of waste, not per unit of recyclable content of the waste. The existence of an interior solution for the recycling effort does not, however, guarantee positive profits for the recycler. In the long run, it is clear that recycler cannot make losses in equilibrium. At the same time, strictly positive profits would lead to entry of new recyclers eroding profit margins for all recyclers. Therefore, the following zero-profit condition is included to ensure a long-term competitive recycling market equilibrium:

$$p_t^w = p_t\beta_t - r(\beta_t) - [1 - \beta_t]p_t^d - t_t^r \qquad (18)$$

This condition ensure that the recycler makes zero profits and at the same time it gives an explicit expression for the market clearing price for waste material. In line with intuiting, the waste price p_t^w will be low if recycled material has low market value p_t, if recycling unit costs $r(\beta_t)$ are high and if landfill costs p_t^d and the recycling tax rate t_t^r are high. Note that the waste price can even become negative, and hence it becomes a waste charge for the consumer, if landfill and recycling costs and taxes would be very high compared to price of the material. In case no recycling would take place (i.e. $\beta_t = 0$), the waste price equals the landfill charge $p_t^w = -p_t^d$ and is passed on completely to the consumer.

Finally, the amount of recycled material that is supplied the material's market is given by:

$$q_t^r = \beta_t w_t \qquad (19)$$

2.2 Market equilibrium, material balance and environmental externalities

With all of the aforementioned equations in mind, we can formulate the market equilibrium for both the material, consumer good and recycling markets. For the consumer good market, consumer demand should equal supply in every period:

$$Q_t = q_t \quad \forall t = 1, 2, \ldots, T \qquad (20)$$

For the materials market, total material demand should equal total supply, which consists of the virgin, substitute and recycled materials that are all assumed to be perfect substitutes:

$$\boldsymbol{q_t = q_t^v + q_t^s + q_t^r \quad \forall t = 1, 2, \dots, T} \tag{21}$$

Finally, we must specify the flow of material throughout the life cycle of the consumption good. It is assumed that material quality does not deteriorate with recycling, so recycled material can be used in the production of new consumption goods, which in turn can be recycled again without incurring quality losses. With regard to the relationship between past consumption and waste generation, the model can be set up in different ways. A first possible way is to assume that goods are not durable and give rise to waste immediately after consumption, with $\boldsymbol{w_t = q_t}$. Packaging of fast moving consumer goods (fruit, vegetable, dairy products) could be an example of this. Alternatively, we can assume that consumption goods only last for one period; this would imply that $\boldsymbol{w_t = q_{t-1}}$. A more general approach is to assume that,

$$\boldsymbol{w_t = \sum_{\tau=1}^{t-1} \phi_\tau q_{t-\tau} \quad \forall t = 1, 2, \dots, T} \tag{22}$$

In equation (22), parameter $\boldsymbol{\phi_\tau}$ represents the breakdown probabilities, which should sum up to one: $\boldsymbol{\sum_{\tau=1}^{T} \phi_\tau = 1}$. This approach is sometimes called the residence time or population balance model (Müller et al., 2014) and different statistical density functions can be used to model the lifetime of the consumption good, like the commonly used bathtub curve for example. Still another option is to set up a relationship between waste and past consumption using a so-called ‘in use stock’ (IUS) or accumulation relationship. In this case, the evolution of the IUS would be modelled as:

$$\boldsymbol{IUS_{t+1} = IUS_t + q_t - w_t \quad \forall t = 1, 2, \dots, T} \tag{23}$$

As can be seen in equation (23), the function is recursive and the IUS in period t consists of all material supplied to the market up to and including period $\boldsymbol{t}$ (inflow). As waste is extracted from the material flow for the purpose of recycling, the corresponding waste volume is deducted from the $\boldsymbol{IUS_t}$ (outflow). Top-down and bottom-up approaches are both used in the literature to quantify the inflow and outflow of material contained in the IUS (Müller et al., 2014). With regard to the waste fraction that becomes available for recycling, it can then be assumed that a particular percentage $\boldsymbol{\alpha}$ of the IUS becomes available for recycling:

$$\boldsymbol{w_t = \alpha IUS_t} \quad (24)$$

When the consumption good is a durable good (i.e. a good that lasts for at least two periods of time), the quantity $\boldsymbol{q_t}$ is to be interpreted as the services the durable good provides to the consumer. Its price $\boldsymbol{p_t}$ is to be interpreted as a rental price for this annual service. This reinterpretation of the model for the consumer would not change the formulas. At the same time however, the production side of the model has to be modified to better capture the link between consumption of services of the durable good, the material embodied in the durable good and its lifetime. An easy way to do this would be to assume that the durable good has a lifetime of $\boldsymbol{l}$ years and that therefore, it takes $\boldsymbol{Q_t/l}$ units of material to provide one unit of service for a year of the consumption good.

As shown in equation (15), only part of the waste that is processed by the recycling plants gets recycled, and the remaining residue is sent to the landfill. Therefore, in the model the volume in the landfills increases according to the following equation:

$$\boldsymbol{LF_{t+1} = LF_t + [1-\beta_t]w_t \quad \forall t = 1,2,\dots,T} \quad (25)$$

In equation (25), parameter $\boldsymbol{LF_t}$ represents the cumulative amount of waste that has been landfilled up to period $\boldsymbol{t}$. We assume in the numerical example in section 3 that landfill capacity is large enough to accommodate the recycling residues. However, the modelling framework can easily be extended to incorporate a landfill capacity constraint $\boldsymbol{LF_t \leq \overline{LF} \quad \forall t = 1,2,\dots,T}$.

Finally, environmental externalities can be linked to different stages of the material flow like the virgin material extraction ($\boldsymbol{q_t^v}$), the recycling process ($\boldsymbol{q_t^r}$) or production of substitute material ($\boldsymbol{q_t^s}$). In addition to flow pollution problems, stock pollution problems can also be modelled; for example, landfills ($\boldsymbol{LF_t}$) causing negative environmental externalities. The framework can also accommodate externalities linked to the use phase of the consumption good ($\boldsymbol{Q_t}$). In general, we write the environmental externalities as follows:

$$\boldsymbol{EXT_t = \varepsilon^v q_t^v + \varepsilon^r q_t^r + \varepsilon^s q_t^s + \varepsilon^{LF} LF_t \; + \varepsilon^Q Q_t} \tag{26}$$

2.3 Monopolist mine owner

In section 2.1, the mining companies or resource owners, recyclers and producers of the substitute material were all assumed to operate in a competitive, decentralised market setting. However, for the mining of virgin material in particular, it is often difficult to maintain the assumption of competitive market behaviour given the high level of market concentration. Therefore, it would be interesting to analyse alternative market structures, in particular monopolistic virgin resource owners. A monopolistic mine owner faces a more complex optimisation problem. First of all, like any monopolist, it can influence the instantaneous equilibrium market price by altering its supply. However, the virgin material residual demand is defined as total market demand minus the demand served by recycled and substitute material. The output choice of the monopolist virgin material supplier influences the material's price, which will also have an effect on recycling efforts being made and substitute material supply possibly. Secondly, a forward-looking monopolist must take into account the impact that its current supply of virgin material has on the availability of waste that forms the input for the recycling industry in subsequent periods. Because derivation of explicit first-order conditions for this scenario is complicated,[6] we programmed an explicit maximisation problem to solve the monopolist's profit maximisation problem, taking into account the supply behaviour of substitute material producers and recyclers, both immediately and in the future. Hence, the profits of the mine owner are defined as the sum of the discounted profit flows:

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[6] See Swan (1980) for an interesting theoretical model of a monopolist anticipating future recycling of its material. Note, however, that this is not a Hotelling-type model but instead focusses on steady-state solutions in the absence of exhaustibility constraints.

$$\boldsymbol{max}_{q_t\,(t=1,2,\ldots,T)}\ \boldsymbol{\pi^v} = \left\{\sum_{t=1}^{T} \boldsymbol{\delta_t}[\boldsymbol{P}(\boldsymbol{Q_t}) - \boldsymbol{c_t^v} - \boldsymbol{t_t^v}]\boldsymbol{q_t^v}\right\} \tag{27}$$

In equation (27), parameter $\boldsymbol{\delta_t}$ still represents the private discount factor. This discount factor might be different from the social discount factor that is used in the first-best welfare scenario below. As the monopolist takes into account the fact that part of the total supply comes from the recycled and substitute material suppliers, the first-order conditions of these alternative suppliers are included in the model as constraints.

2.4 First-best welfare optimisation

Apart from the market scenarios defined above, we also consider a welfare optimisation scenario. In order to be able to formulate the first-best welfare optimisation problem, we must first define the social welfare function. In the model, social welfare is defined as the sum of utility minus the production costs of the virgin, substitute and recycled material suppliers and the cost of all environmental externalities during the entire lifetime of the good. Taxes and subsidies are left out of this equation, as these are just redistributions of income and profits. This gives us following equation, with variable $\boldsymbol{W}$ representing welfare:

$$\boldsymbol{W} = \sum_{t=1}^{T} \widetilde{\boldsymbol{\delta}}_{\boldsymbol{t}}\,[\boldsymbol{U_t} - \boldsymbol{c_t^v q_t^v} - \boldsymbol{c_t^s q_t^s} - \boldsymbol{r}(\boldsymbol{\beta_t})\boldsymbol{w_t} - \boldsymbol{EXT_t}] \tag{28}$$

Note that in equation (28), a social discount factor ($\widetilde{\boldsymbol{\delta}}_{\boldsymbol{t}}$) is used instead of the private discount factor $\boldsymbol{\delta_t}$. In practice, companies often employ a higher discount rate than social planners because they account for risk and are under pressure from their investors to deliver short-term returns (Jagannathan et al., 2016). According to the Hotelling rule, the higher discount rate implies a more rapid exhaustion of a non-renewable resource stock, leaving less for future generations. In turn, this implies that remaining resource stocks are exploited at a faster rate than is desirable from a social welfare point of view.

In addition to the social discount factor, equation (28) takes into account externalities that arise at different stages of the materials’ life cycle (virgin material extraction, recycling, landfilling).

These externalities are represented by parameter $\boldsymbol{EXT_t}$ and were defined in expression (26) above.

3 Illustrative simulations

In order to illustrate the generic applicability of the model, this chapter elaborates on a numerical example and shows typical outcomes and results that can be generated based on the theoretical underpinnings presented in the previous chapter.

3.1 Case description

The input parameters used in this chapter are based on a realistic case in which a non-renewable resource is extracted and used in the production of a consumption good. To make the descriptions more clear and intelligible, we refer to this non-renewable resource as sand.

Given knowledge of the different equations presented in section 2, together with the different case input parameters shown in Appendix A, it is possible to obtain example results with respect to initial, interim and final market prices; shadow prices; recycling efforts; and supplied volumes of virgin, substitute and recycled sand. The time period of reserve exhaustion $\boldsymbol{T}$ is unknown and is treated as an endogenous variable. For the illustrative simulations we chose to consider only one externality which is linked to the stock of all landfilled material because policy interventions are typically more complex in the case of stock externalities compared to the case of flow externalities linked to annual production or consumption rates. To resolve the optimisation problems, GAMS modelling software[7] was used, in line with previous studies (Caplan, 2004; Conrad, 1999; Flakowski, 2004). For this GAMS implementation, a mixed complementarity program (MCP) format was adopted to accommodate for the non-negativity restrictions in the consumers', virgin and substitute material producers' maximization problems. By using first-

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[7] General Algebraic Modeling System, see https://www.gams.com for details.

order conditions to set up the model, the main advantage of this kind of formulation lies in its flexibility and speed in solving complex economic models[8].

3.2 Simulation results: reference case (R)

Figure 1 shows the evolution over time of the consumer price for three different scenarios: perfectly competitive markets (competition), monopoly in virgin material production (monopoly) and first-best welfare optimum (first best). The consumer price is the net price the consumer faces, i.e. the resource price plus consumption tax minus the waste price: $\boldsymbol{p_t + t_t^q - p_t^w}$. In the competitive scenario (dashed line), the consumer price of sand increases from 5.45 euro/ton to 12 euro/ton, which is equal to the choke-off price level. Figure 2 shows the evolution of the market price $\boldsymbol{p_t}$ which is the price the producer of the virgin material and the recyclers receive when they sell material. As we assumed in the simulations that the marginal cost of mining sand is constant, the market price in the competitive scenario follows the Hotelling rule. The shadow price $\boldsymbol{\lambda_t = p_t - c_t^v - t_t^v}$ increasing over time at the assumed private rate of discount of 3%. It takes 57 periods before the virgin sand reserve is completely exhausted. In the first-best welfare optimal scenario however, the optimal time of depletion of the resource is 71 periods. The difference with the competitive market outcome is due to the fact that we assumed an externality cost of 0.10 euro per ton caused by the accumulation of material in the landfill. This externality raises the social cost of sand extraction and therefore calls for a slower welfare optimal production rate compared to the competitive market scenario without taxes.

Looking at the monopoly scenario (dotted line), the figures demonstrate that the monopolist will restrict output, resulting in a market and consumer price that is initially higher than in the competitive market scenario. However, the rate of price increase is slower which leads to a substantial increase in the time horizon over which the sand is extracted. In the monopoly

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[8] The GAMS code used for our simulations is available from the authors upon request.

scenario it takes 91 periods to fully deplete the initial virgin sand reserve. Although the monopolist mitigates the scarcity issue, it is important to realise that market power may lead to substantive welfare losses. This is confirmed by the welfare figures shown in Table 1 below. Monopoly leads to the worst welfare outcome in our illustrative simulation because the welfare losses of monopoly supply behaviour are higher than the welfare gain from postponing the date of exhaustion. Note also that the monopolist is capable of claiming a much larger share of the total welfare. Compared to the competitive market scenario, profits of the virgin material producer are more than 20% higher and the consumer surplus is almost 50% lower.

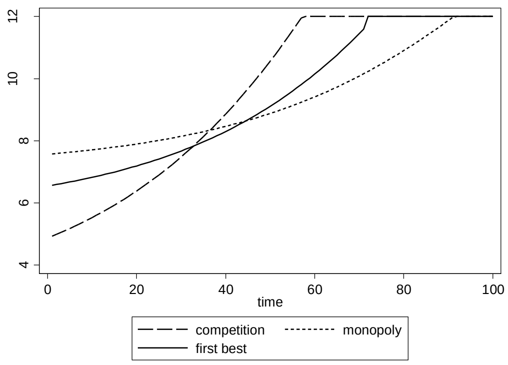

Figure 1: Consumer price evolution

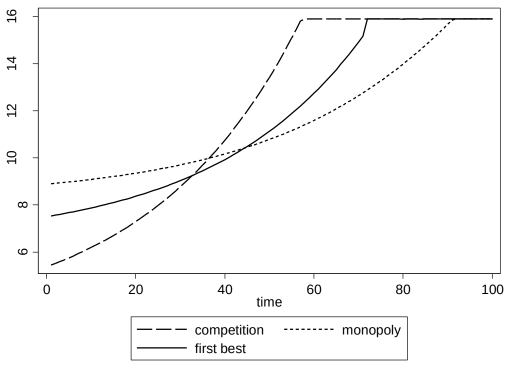

Figure 2: Market price evolution

Figures 3 shows the evolution over time of the supply of virgin material $\boldsymbol{q_t^v}$. The corresponding evolution of the remaining stock of virgin material $\boldsymbol{S_t}$ is depicted in Figure 4. As predicted by the Hotelling rule, the supply of virgin material decreases over time and reaches zero after 57 period in the competitive market scenario. In contrast, the monopolist spreads its extraction activities more over time deferring the time of exhaustion of the virgin sand reserve until period 91. The first best welfare optimal extraction path of virgin material lies in between the competitive and monopoly path. Finally, note that the supply of substitute material (not shown) is zero in this simulation. This is a consequence of the fact that, in this particular model simulation, the cost of supplying substitute material is higher than the choke-off price (50 euro per ton versus 12 euro per ton). As a result, the substitute never comes into the market, also not after exhaustion of the domestic reserves of virgin sand.

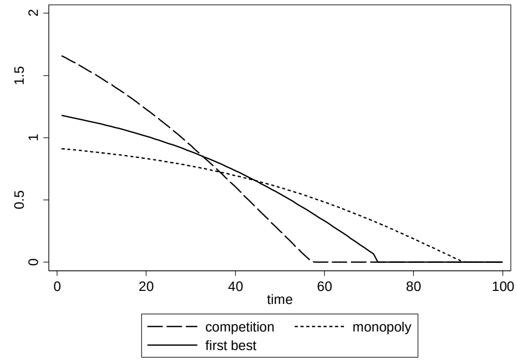

Figure 3: Evolution of supply of virgin material

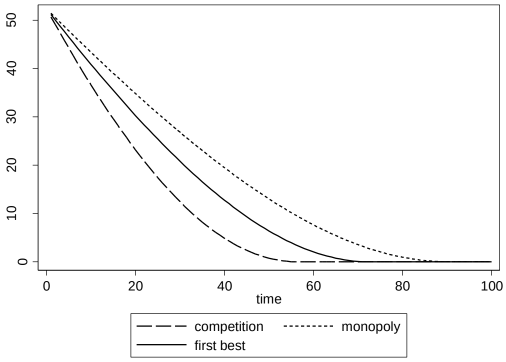

Figure 4: Evolution of remaining stock of virgin material

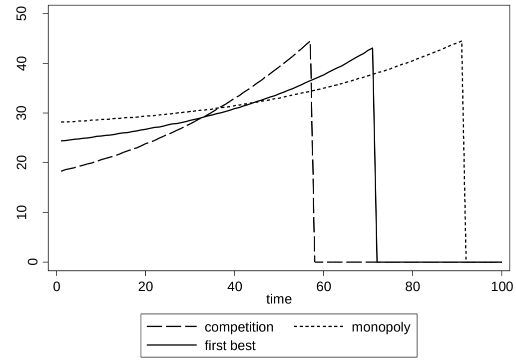

Figure 5: recycling rate

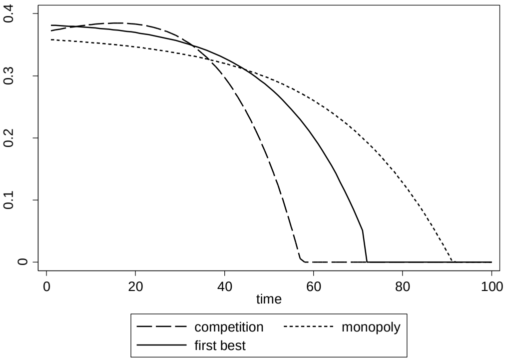

Figure 6: quantity recycled material

Figure 5 shows the evolution over time of the recycling efforts $\boldsymbol{\beta_t}$ that the price-taking recyclers choose to maximise their profits. Recall from first-order condition (16) that the recycling effort is driven by the market price of the material $\boldsymbol{p_t}$. Hence, the evolution of recycling efforts and the ranking over scenario's is the same as in Figure 2. The highest recycling rate of 44.6% is reached as the market price reaches its maximum of 15.89 euro per ton. As Figure 5 shows, the monopoly scenario generates the highest recycling efforts initially. This might seem counter intuitive as recycled material competes with virgin material and one would think the monopolist would try to limit recycling efforts in order to protect its dominant market position. However, as recycling efforts are driven by the market price of material, the monopolist has to balance two counteracting forces. On the one hand the monopolist wants to increase the market price to enjoy higher revenue. But on the other hand, higher market prices lead to more recycling and erosion of the monopolist's market power. Figure 6 shows the evolution of the quantity of recycled material coming to the market, i.e. $\boldsymbol{q_t^r} = \boldsymbol{\beta_t w_t}$. It shows that initially, the monopoly supply of recycled material is lower than in the competitive and in the first best scenario in spite of the higher recycling effort. This is due to the lower amount of virgin material, and hence waste, that becomes available for recycling under monopoly. Eventually however, more recycled material is produced in the monopoly scenario compared to the other scenario's. The surface under the recycled material supply curve in monopoly is higher than under the competitive and first best scenario.

Table 1 summarises some key numbers that characterize the base case simulation. As expected, total discounted welfare is highest in the first best scenario and total discounted profits of the virgin material producer are highest in the monopoly scenario. Table 1 also confirms that the total sum of recycled material is highest in the monopoly scenario (recall Figure 6). Perhaps surprising, total discounted externality costs are lowest in the monopoly scenario. In our reference case simulation, we only considered an externality linked to the landfill. In the end, all scenarios lead to the same quantity of landfilled material. Because of material balance, all virgin material eventually ends up in the landfill but the time path is different because of the differences in extraction rate and recycling in the different scenarios. The reason that the monopoly scenario leads to the lowest discounted externality costs is due to the fact that it is also the scenario with the slowest accumulation rate of the landfill. The externality costs are increasing more slowly and because of the discounting, the later time periods add relatively less to the sum of discounted externality costs.

Table 1: Key statistics reference case simulation (R)

variable	Competition	Monopoly	First best
T (periods)	57	91	71
$\sum \boldsymbol{q}_t^v$ (10^6 ton)	52.324	52.324	52.324
$\sum \boldsymbol{q}_t^r$ (10^6 ton)	17.507	24.274	21.401
$\sum \boldsymbol{q}_t^s$ (10^6 ton)	0	0	0
$\sum \boldsymbol{q}_t$ (10^6 ton)	69.831	76.598	73.725
Total discounted consumer surplus[9] (10^6 euro)	120.483	66.891	88.240
Total discounted profits virgin material producer (10^6 euro)	128.050	157.255	152.051
Total discounted externality costs (10^6 euro)	100.479	72.568	85.022
Total discounted welfare[10] (10^6 euro)	148.054	151.577	155.270

Consumer surplus, virgin material producer's profits, externality costs and welfare are calculated over the full time horizon of 100 years and are discounted using the social rate of discount.

3.3 Simulation results: sensitivity analyses

We now present four variations on the parameters of the reference simulation. We first consider a scenario with lower costs of substitute material. A second sensitivity analysis introduces a gap between the private and social discount rate. The third sensitivity considers a tax on disposal of recycling residues, in other words a landfill tax. In the fourth and last sensitivity analysis we consider a revenue neutral combination of a tax on virgin material extraction with a subsidy for recycling.

Sensitivity analysis 1: lower cost of substitute material (S1).

In the reference scenario, substitute material does not come to the market because the cost of supplying it is higher than the choke-off price. In terms of the competitive scenario, this situation is represented in Figure 7. The light grey area represents the amount of virgin sand extraction and the darker grey area represents the amount of recycled material. After 57 periods no sand

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[9] Consumer surplus in period $\boldsymbol{t}$ is the difference between utility and the expenditure of the consumer: $\boldsymbol{U}(\boldsymbol{Q}_t) - [\boldsymbol{p}_t + \boldsymbol{t}_t^q]\boldsymbol{Q}_t + \boldsymbol{p}_t^w \boldsymbol{w}_t$. The discounted sum of this consumer surpluses over the entire time horizon is reported in the table.

[10] Note that total welfare is always equal to the sum of consumer surplus, producers' profits (which are zero for the producers of the substitute material and for the recyclers because we assume perfect competition in these sectors), externality costs and government tax revenues (if relevant).

comes to the market anymore because there is no waste to be recycled and because the substitute is too expensive compared to the marginal willingness to pay of the consumers.

Figure 8 depicts the sensitivity scenario S1 with lower substitute material marginal production costs of 10 euro ton. In that scenario, the material price hits 10 euro per ton in period 41 after which the substitute supply takes over the market. When the substitute comes onto the market, its marginal production cost acts as a new choke-off price resulting in a switch in supply from virgin to substitute material. The virgin material reserve is completed exhausted by that time (see Table 2).

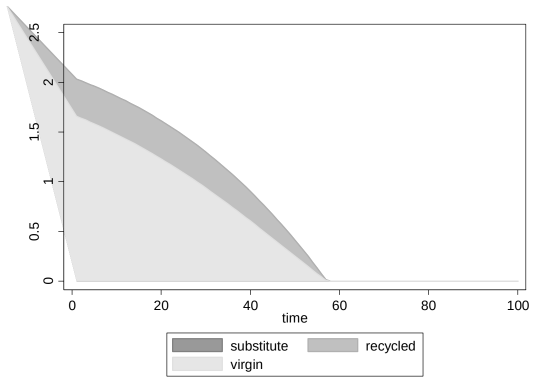

Figure 7: quantities of materials in competition scenario (R)

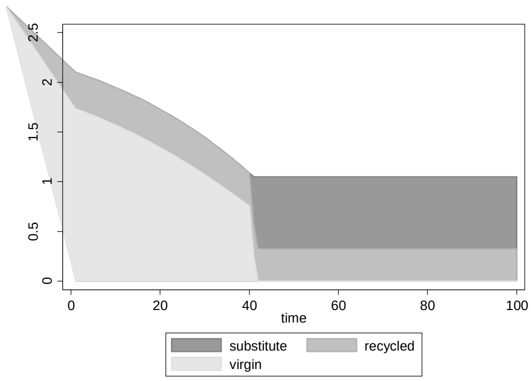

Figure 8: quantities of materials in competition scenario (S 1)

Table 2 shows how the simulation results change when the substitute material makes it to the market. Compared to the reference scenario, substiantially higher amounts of material are produced and consumed. To a large extent, this is the result of the steady influx of the substitute material in the long run. Whereas no sand was consumed after exhausting domestic reserves in the reference scenario, a new and seemingly unlimited source of substitute material serves the market after exhaustion of domestic virgin reserves. Note that the difference in discounted externality costs is not so pronounced between the reference and the sensitivity scenario. This is at first sight surprising as much more material is consumed which eventually ends up in the landfill. As this effect is only playing in the very long run, the difference in externality cost is strongly diminished because of the discounting formula.

Table 2: Key statistics sensitivity scenario S1

variable	Competition	Monopoly	First best
T (periods)	41	69	61
$\sum \boldsymbol{q_t^v}$ (10^6 ton)	52.324	52.324	52.324
$\sum \boldsymbol{q_t^r}$ (10^6 ton)	34.451	33.207	30.965
$\sum \boldsymbol{q_t^s}$ (10^6 ton)	43.125	23.467	19.846
$\sum \boldsymbol{q_t}$ (10^6 ton)	129.899	108.997	103.135
Total discounted consumer surplus (10^6 euro)	149.965	74.200	95.026
Total discounted profits virgin material producer (10^6 euro)	112.282	155.093	150.134
Total discounted externality costs (10^6 euro)	118.856	76.051	89.300
Total discounted welfare (10^6 euro)	143.391	153.242	158.683

Consumer surplus, virgin material producer's profits, externality costs and welfare are calculated over the full time horizon of 100 years and are discounted using the social rate of discount.

Sensitivity analysis 2: private discount rate exceeding social discount rate (S2)

We now consider the case in which the private discount rate is raised to 6 per cent, while the social discount rate still being equal to 3 per cent. As the same social discount rate applies as in the reference simulations, the results for the first best scenario are exactly the same as in the reference simulation. In the competitive scenario however, the higher private discount rate results in the equilibrium price path having a steeper slope than before. This is a logical consequence of the rise in the private discount rate as, according to the Hotelling rule, competitive market equilibrium prices grow at the private market interest rate. The steeper slope means that the choke-off price level is reached more quickly than before. This implies that the time interval in which virgin sand is mined is shorter than before and equals now 43 periods versus 57 in the reference simulation. The same reasoning applies in the monopolistic scenario. The time horizon over which the sand is extracted is still longer than in the competitive scenario but shorter than before and equals 73 periods versus 91 before. Note that the monopoly scenario now comes very close to the first best scenario. For the parameter values chosen in this simulation, both scenarios result in very similar extraction paths.

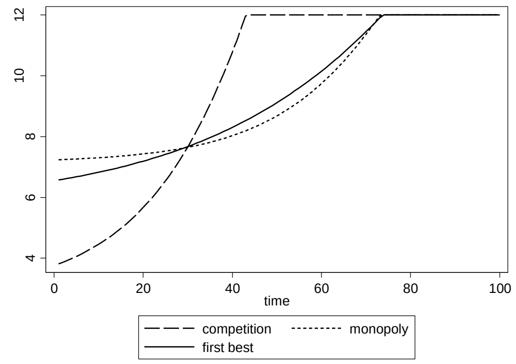

Figure 9: Consumer price evolution (scenario S2)

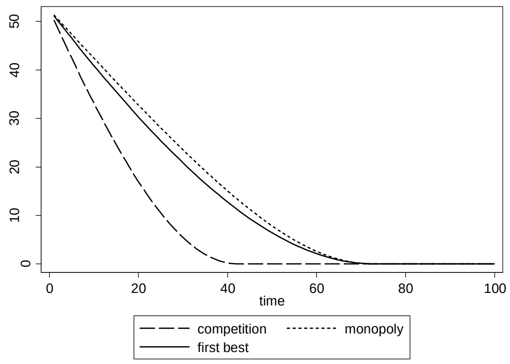

Figure 10: Evolution of remaining stock of virgin material (scenario S2)

Table 3: Key statistics sensitivity scenario S2, S3 and S4

variable	Competition (S2)	Monopoly (S2)	Competition (S3)	Competition (S4)
T (periods)	43	73	70	60
$\sum \boldsymbol{q}_t^v$ (10^6 ton)	52.324	52.324	52.324	52.324
$\sum \boldsymbol{q}_t^r$ (10^6 ton)	13.733	22.035	20.874	21.485
$\sum \boldsymbol{q}_t^s$ (10^6 ton)	0	0	0	0
$\sum \boldsymbol{q}_t$ (10^6 ton)	66.057	74.359	73.198	73.809
Total discounted consumer surplus (10^6 euro)	149.454	79.392	92.585	124.869
Total discounted profits virgin material producer (10^6 euro)	93.050	154.987	65.392	122.659
Total discounted externality costs (10^6 euro)	111.629	79.972	87.317	97.992
Total discounted tax revenue (10^6 euro)	—	—	84.464	-0.244
Total discounted welfare (10^6 euro)	130.875	154.407	155.125	149.292

Consumer surplus, virgin material producer's profits, externality costs and welfare are calculated over the full time horizon of 100 years and are discounted using the social rate of discount.

Sensitivity analysis 3: introducing a levy on landfilling (S3)

We now examine whether we can replicate in a competitive market setting the first-best outcome by introducing an appropriate landfill tax. Recall that the landfill tax or price for disposal of recycling residues $\boldsymbol{p_t^d}$ has an impact on recycling efforts (through first order condition (16)) and on the waste price and therefore on consumption (through first order condition (3)). After testing for several values, we present here the case of $\boldsymbol{p_t^d} = 3$ euro per ton in every time period. As can be seen from comparing the last column of Table 3 with the last column of Table 1, this rate of landfill tax makes the competitive market outcome replicate closely the first best welfare maximizing outcome. A noticeable characteristic of this simulation is also that the landfill pricing causes the waste price $\boldsymbol{p_t^w}$ to be negative most of the time as illustrated by the dotted line in Figure 11 below. Hence this implies that the consumer has to pay most of the time for disposing of end of life consumption goods. This simulation illustrates the flexibility of the modelling framework as it can accommodate different real world situations on actual waste markets where sometimes waste is valuable (think of many types of scrap metal) and in other cases it is a costly burden (think of many types of hazardous waste).

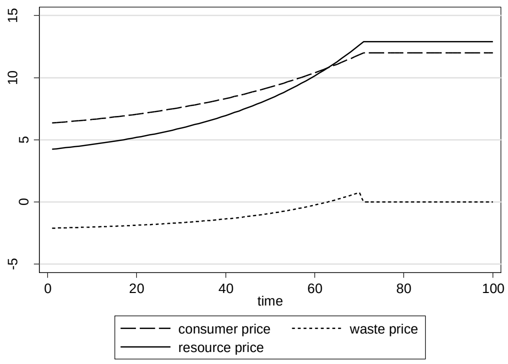

Figure 11: Consumer price evolution (scenario S2)

Sensitivity analysis 4: a tax on virgin material extraction combined with a recycling subsidy (S4)

The final sensitivity analysis we present is a combination of a virgin material extraction tax ($\boldsymbol{t_t^v} = 1.5$) and a subsidy for recycling (negative tax on recycling activity $\boldsymbol{t_t^r} = -1.1$). The rationale for this simulation is the following. First, we have learned from previous simulations that without intervention, the competitive market scenario results in too fast depletion of the virgin material reserves. Taxing virgin material extraction is probably an effective way to counter this effect. Secondly, stimulating recycling could prolong the time that material is used in the economy and hence, it could contribute to alleviate material scarcity by boosting supply of an alternative source of material. We also want to make the combination of tax and subsidy revenue neutral for the government as this is politically often easier to implement than pure subsidy or tax schemes. The key statistics of this simulation are presented in column S4 of Table 3 higher. As can be seen, the scenario defers depletion compared to the unchecked competitive market scenario ($\boldsymbol{T} = 60$ instead of 57) but it fails to achieve the first-best horizon of 71 periods. A similar conclusion prevails regarding social welfare. The tax-subsidy combination improves marginally over the unregulated competitive market scenario but it falls short of the first-best outcome. The reason why this combination of a virgin extraction tax and recycling subsidy does not work well has to do with the distorting effect of the recycling subsidy. The subsidy increases the waste price that consumers' receive from the recyclers and hence it lowers the consumption price. Therefore, consumers are inclined to consume more compared to a simulation without recycling subsidy. This simulation illustrates that the modeling framework can be used to evaluate the effect of a combination of policy instruments on key variables as welfare, externality costs and the distribution of welfare over the consumers and producers.

4 Discussion of capabilities and limitations of the model

Because of its generic design, the modelling framework presented in this paper can be of great value to policy makers when designing and fostering sustainable practices for all sorts of non-renewable resources. Input parameters can be adapted to reflect different characteristics like technologies, remaining reserves, costs, environmental externalities etc. Also, appropriate formulations can be used to simulate competitive or monopolistic market outcomes, and first-best welfare optimisation scenarios including environmental externalities in the extraction, production, recycling, consumption or waste disposal phase of the material's life cycle. This flexible framework allows to (i) identify welfare optimal outcomes and (ii) investigate market outcomes under different combinations of subsidy and tax instruments. In particular, policy makers can use the framework to fine tune policy instrument mixes in order to steer behaviour towards the social welfare optimizing levels. At the same time however, we should warn against too high expectations about the accuracy of the model results for setting tax and subsidy rates in the real world. As all models, our model is based on often heroic assumptions regarding behaviour of agents (utility and profit maximization), market structure (perfect competition or monopoly), information availability (perfect information and no uncertainty) and data sources (private production cost data). For every real world application, the appropriateness of the assumptions and quality of data input have to be judged carefully when interpreting the simulation results.

Although the flexible modelling framework adds significantly to the existing literature, we are well aware of its limitations, many of which offer interesting possibilities for future research. We believe that the most important of these limitations are the following. First, the model could be expanded to allow for different jurisdictions that are capable of setting their own policy instruments, in order to maximise their domestic welfare. Such a model could be used to investigate the international policy competition, perhaps leading to a “race to the bottom” in externality taxes or “race to the top” in minimum recycling rates. Secondly, in the current version of the model, producers of the consumption goods only choose production volumes and cannot adjust quality aspects of their goods, such as longevity, material intensity and green design or design for recycling. Allowing for a more realistic set of choices for producers would definitely enrich the model. Thirdly, we assumed a perfectly competitive market for recyclers. Although this is often the case in reality (think of small scale independent steel mills that use scrap metal or aluminium remelters), it is clear that also recycling markets might be dominated by only a few and strategically behaving companies. In particular, they might want to lower the price of waste they buy from consumers, or to increase the price of recycled material. Relaxing the perfect competition assumption in the recycling market is interesting but technically challenging because of the possible interference with the monopolistic virgin material producer. Fourthly, we assumed so far that virgin material producers and recyclers are independent companies each maximizing their own individual profits. Other settings are conceivable in which virgin material producers and recyclers are vertically integrated and maximizing joint profits. Fifthly, it would be interesting to allow for more complex consumers’ behaviour including illegal waste disposal, leasing instead of buying goods or to include a second-hand market of older vintage goods. Sixthly, it might in some situations and regions be relevant to include in the model an upper limit on the landfill capacity. If binding, that would introduce another type of scarcity in the model and would lead to an increasing landfill price over time. The numerical model can easily be extended to accommodate such a constraint. Finally, it could be interesting to take a closer look at the effects of recycling on material quality deterioration and to allow for recycled materials being only an imperfect substitute for virgin materials.

Many of these extensions have been studied separately in the literature using only theoretical and analytical models. Incorporating these extensions in our numerical simulation modelling framework will add considerable complexity to the model. We are however convinced that only by using a consistent numerical simulation modelling framework, such as the one we have presented in this paper, will it be possible to investigate combinations of these extensions in more complicated but realistic scenarios.

5 Conclusions

Debates on supporting the transition towards a more resource-efficient and low-carbon economy have focused on how to identify optimal extraction paths over time for any particular non-renewable resource reserve. This paper adds to the literature by developing a generic numerical optimisation model that can be used to simulate the effects that different policy instruments can have within the material flow of a particular non-renewable resource. The modelling framework is flexible to allow for different assumptions regarding behaviour of market participants (profit maximisation in a competitive or monopolistic market setting) and to be capable of comparing decentralised market-based scenarios with social welfare maximising scenarios that take into account environmental externalities at various stages of the material's life cycle.

By using a fixed initial non-renewable resource reserve, a cake-eating model was built, similar to the well-known Hotelling model. Several extensions were added that, to our knowledge, had never previously been combined together with such a Hotelling model. The first extension relates to the inclusion of a recycling sector in which recyclers choose a recycling effort in order to maximise profits. Consequently, recycling is an endogenously defined function within the optimisation model. The recyclers source input for their recycling process on a waste market where consumers try to dispose of end-of-life consumption products. The second extension is that we allow for the possibility that a substitute material can come onto the market at a fixed price. If such a substitute – such as imported material from abroad – came on the market, its price would act as a choke-off price at which the switch is made from virgin to substitute material. This substitute would actually constitute a third supply source, next to virgin and recycled material. Throughout the developed model, the full material flow system that includes these different supply sources is taken into account by imposing appropriate material balance constraints. As recycling rates will never reach 100%, every unit of material will, eventually, end up as recycling residue in a landfill. Thirdly, environmental externalities are considered that can be linked to different stages of the material's life cycle. We distinguish between externalities caused by the production of virgin and substitute material, by the recycling process, by the consumption phase of the good, or by the accumulation of recycling residues in the landfill. Fourthly, we introduced different policy instruments (extraction, production or consumption taxes, waste taxes, etc.) that can be used to correct for different environmental externalities. Fifthly, different degrees of product durability can be simulated by selecting different functional relationships between past consumption and future waste generation.

As the various simulation examples and sensitivity analyses have shown, the results are all in line with expectations based on theoretical insight and intuition. This indicates that the model is able to produce meaningful results that are based on a well-founded, realistic and stable methodological structure. In addition, the model is capable of quantifying effects that are very hard to assess in purely analytical and theoretical models. An example is the impact on market prices, recycling efforts and the date of exhaustion of virgin material reserves in the case of a farsighted monopolist producer of virgin material who anticipates future recycling of the waste containing the material that he or she brings to the market today. Also the model can be used to assess the combined impact of different tax and subsidy instruments at different life cycle stages. This is particularly interesting for policy makers as it allows them to fine tune realistic packages of multiple policy instruments. For example, we have shown firstly that a constant landfill tax can be used to approximate very closely the first-best welfare optimal outcome in terms of externality costs and reserve exhaustion date. Secondly, we have illustrated that a government revenue neutral combination of a tax on the extraction of virgin material and a subsidy on recycling activity cannot easily replicate the first-best outcome. The recycling subsidy distorts the waste price and gives false signals to consumers regarding the social cost of consumption.

Appendix A: input data simulations

Input parameters	
a_t: intercept inverse demand function, choke-off price	12
c_t^v: marginal cost virgin production	3
c_t^s: marginal cost substitute production	50 [S1: 10]
t_t^q: tax on consumption	0
t_t^v: tax on virgin material production	0 [S4: 1.5]
t_t^r: tax on recycled material production	0 [S4: –1.1]
t_t^s: tax on substitute material production	0
p_t^d: price for disposal of recycling residues	0 [S3: 3]
S_0: Initial resource stock at time zero (10^6 ton)	52,324
IUS_0: Initial in use stock at time zero	0
LF_0: Landfilled waste volume at time zero	0
ρ: private discount rate	0.03 [S2: 0.06]
$\widetilde{\rho}$: social discount rate	0.03
ε^v: marginal external cost of virgin material production	0
ε^s: marginal external cost of substitute material production	0
ε^r: marginal external cost of recycled material production	0
ε^{LF}: marginal external cost of stock of landfilled material	0.1
ε^Q: marginal external cost of consumption	0
Calibrating parameters	
b_t: absolute value slope of inverse demand function	6/1,724,000
g_t: starting value marginal recycling cost function parameter	6/log(1-0.20)

All quantity variables and parameters (initial stocks) are in million ton. All monetary variables and parameters (marginal production costs, prices, taxes, marginal external costs) are in euro per ton.

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